SECTION B

Q5. (a) Obtain the partial differential equation by eliminating the arbitrary function f from the equation $f(x + y + z,\ x^2 + y^2 + z^2) = 0$. *8*

(b) Obtain the following approximate quadrature formula :

$$\int_0^3 f(x)\,dx = \frac{3}{8}\,[f(0) + 3f(1) + 3f(2) + f(3)].$$ *8*

(c) (i) Convert $(523.0234375)_{10}$ into an equivalent octal number and then convert it to its binary form.

(ii) If $x = (1D2.2)_{16}$ and $y = (52E.02)_{16}$, then find the value of x + y in decimal system. *4+4*

(d) The velocity components in an unsteady three dimensional flow are given by $u = \frac{x}{1+t}$; $v = \frac{y}{1+t}$, $w = \frac{z}{1+t}$. Describe the streamlines and pathlines. *8*

(e) Is a system of two particles which are connected by a rod of constant length holonomic ? Justify your answer. *8*

Q6. (a) Using Charpit's method, find the complete integral of $yq + 3xp = 2(z - y^2p^2)$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. *10*

(b) Write down the algorithm for solving the differential equation $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, numerically by Euler's method with step length h up to $x = x_n = x_0 + nh$.

Solve the following differential equation for x = 1 with step length h = 0·2 by using Euler's method :

$$\frac{dy}{dx} = x^2 + y,\ y(0) = 1.$$ *6+9*

(c) Derive the Hamilton equations for holonomic systems and use them to discuss the motion of a simple pendulum. *15*

Q7. (a) Show that the iteration formula for the Newton-Raphson method for finding the K^{th} root of a positive real number a is :

$$x_{n+1} = \frac{1}{K}\left[(K-1)x_n + \frac{a}{x_n^{K-1}}\right], \text{ where } K > 0.$$

Use this formula to find $\sqrt[3]{13}$, correct up to three decimal places. *3+7*

(b) Find the general solution of the partial differential equation

$[D^2 - (D')^2 - 3D + 3D']z = (1-x)(1-y) + e^{x+2y}$,

where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. *15*

(c) Consider an inviscid incompressible fluid flow with velocity $\bar{q} = \left(x, \frac{y}{1+t}, \frac{z}{2+t}\right)$ under the body force $\bar{F} = -gz\hat{k}$, where g is the gravitational constant. Find the pressure at a point (x, y, z) if $p(0, 0, 0) = p_0$. *15*

Q8. (a) Consider a source and a sink of equal strength at points $\left(\pm\frac{1}{4}a, 0\right)$ within a fixed circular boundary $x^2 + y^2 = a^2$. Determine the equation of streamlines. *10*

(b) Find the solution of the heat equation

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \; 0 < x < \pi, \; t > 0$$

under the boundary conditions $u(0, t) = 0 = u(\pi, t)$ and the initial condition

$$u(x, 0) = \begin{cases} x, & 0 \le x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi. \end{cases}$$ *15*

(c) Use Gauss-Jordan elimination method to solve the following system of equations :

$$3x_1 + x_2 + x_3 = 7$$

$$2x_1 + x_2 + 5x_3 = 13$$

$$x_1 + 4x_2 + x_3 = 9 \cdot 4$$

correct up to 2-significant figures. *15*