IAS
PREVIOUS YEARS QUESTIONS (2020-1983)
SEGMENT-WISE

VECTOR ANALYSIS

2020

❖ For what value of a, b, c, is the vector field $\bar{V} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$ irrotational ? Hence, express $\bar{V}$ as the gradient of a scalar function ϕ. Determine ϕ. **[10]**

❖ For the vector function $\bar{A}$, where $\bar{A} = (3x^2 + 6y)\hat{i} - 14yz\,\hat{j} + 20xz^2\,\hat{k}$, calculate $\oint_C \bar{A}\cdot d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths :

(i) $x = t, y = t^2, z = t^3$

(ii) Straight lines joining (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1)

(iii) Straight line joining (0, 0, 0) to (1, 1, 1)

Is the result same in all the cases ? Explain the reason. **[15]**

❖ Verify the Stokes' theorem for the vector field $\bar{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ on the surface S which is the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1$, $-2 \le y \le 2$; S is oriented upwards. **[20]**

❖ Evaluate the surface integral $\iint_S \nabla \times \bar{F}\cdot \hat{n}\,dS$ for $\bar{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **[15]**

2019

❖ Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$ and $z = t^3$ at the point (1, 1, 1). **[10]**

❖ Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). **[15]**

❖ Find the radius of curvature and radius of torsion of the helix $x = a\cos u$, $y = a\sin u$, $z = au\tan\alpha$. **[15]**

❖ State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **[15]**

❖ Evaluate by Stoke's theorem $\oint_C e^x\,dx + 2y\,dy - dz$, where C is the curve $x^2 + y^2 = 4$, $z = 2$. **[05]**

2018

❖ Find the angle between the tangent at a general point of the curve whose equations are $x = 3t$, $y = 3t^2$, $z = 3t^3$ and the line $y = z - x = 0$. **(10)**

❖ If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate $\iint_S [(x+z)dydz + (y+z)dzdx + (x+y)dxdy]$ using Gauss' divergence theorem. **(12)**

❖ Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{i} + a(1 - \cos u)\vec{j} + bu\,\vec{k}$ **(12)**

❖ Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$. Show that

$$\text{curl}(\text{curl}\,\vec{v}) = \text{grad}(\text{div}\vec{v}) - \nabla^2\vec{v}.$$ **(12)**

❖ Evaluate the line integral $\int_C -y^3dx + x^3dy + z^3dz$ using Stoke's theorem. Here C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. The orientation on C corresponds to counterclockwice motion in the xy-plane. **(13)**

❖ Let $\vec{F} = xy^2\vec{i} + (y + x)\vec{j}$. Integrate $(\nabla \times \vec{F})"\vec{k}$ over the region in the first quadrant bounded by the curves $y=x^2$ and $y=x$ using Green's theorem. **(13)**

2017

❖ For what values of the constants a, b and c the vector $\vec{V} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. **(10)**

❖ The position vector of a moving point at time t is $\vec{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time t = 0. **(10)**

❖ Find the curvature vector and its magnitude at any point $\bar{r} = (\theta)$ of the curve $\bar{r} = (a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **(16)**

❖ Evaluate the integral : $\iint_S \bar{F}\cdot\hat{n}ds$ where $\bar{F} = 3xy^2\hat{i} + \left(yx^2 - y^3\right)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \le 4, -3 \le x \le 3$, using divergence theorem. **(09)**

❖ Using Green's theorem, evaluate the $\int_C F(\bar{r})\cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = \left(x^2 + y^2\right)\hat{i} + \left(x^2 - y^2\right)\hat{j}$ and $d\bar{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \left\{(x, y) \mid 1 \le y \le 2 - x^2\right\}$. **(08)**

2016

❖ Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. **(10)**

❖ Find f(r) such that $\nabla f = \frac{\vec{r}}{r^5}$ and f(1) = 0. **(10)**

❖ Prove that

$$\oint_C f\, d\vec{r} = \iint_S d\vec{S}\times\nabla f$$ **(10)**

❖ For the cardioid r = a (1 + cosθ), show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if θ = 0, À/4, À/2. **(15)**

2015

❖ Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z = x^2+y^2-3$ at (2, –1, 2). **(10)**

❖ Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2y+z^3=4$ may intersect orthogonally at (1, –1, 2). **(12)**

❖ A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$

Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. **(12)**

❖ Evaluate $\int_C e^{-x}(\sin y\,dx + \cos y\,dy)$, where C is the rectangle with vertices (0, 0), (π, 0), $\left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$. **(12)**

2014

❖ Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\,\hat{i} + t\sin t\,\hat{j}, 0 \le t \le 2\pi$. Give its magnitude also. **(10)**

❖ Evaluate by Stokes' theorem

$$\int_\Gamma (y\,dx + z\,dy + x\,dz)$$

where Γ is the curve given by

$x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$, starting from (2*a*, 0, 0) and then going below the *z*-plane. **(20)**

2013

❖ Show that the curve

$$\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$$

lies in a plane. **(10)**

❖ Calculate $\nabla^2(r^n)$ and find its expression in terms of *r* and *n*, *r* being the distance of any point (*x,y,z*) from the origin, n being a constant and ∇^2 being the Laplace operator. **(10)**

- A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. **(10)**
- By using Divergence Theorem of Gauss, evaluate the surface integral $\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-1/2} dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. **(15)**
- Use Stokes theorem to evaluate the line integral $\int_C \left(-y^3dx + x^3dy - z^3dz\right)$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x + y + z = 1$. **(15)**

2012

- If $\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$
  $\vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$
  Find the value of $\frac{\partial^2}{\partial x \partial y}\left(\vec{A}\times\vec{B}\right)$ at (1, 0, –2). **(12)**
- Derive the Frenet-Serret formulae.
  Define the curvature and torsion for a space curve. Compute them for the space curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$
  Show that the curvature and torsion are equal for this curve. **(20)**
- Verify Green's theorem in the plane for $\oint_C \left\{\left\{xy + y^2\right\}dx + x^2dy\right\}$ where C is the closed curve of the region bounded by y = x and y = x^2. **(20)**
- If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate $\iint_S \left(\vec{\nabla}\times\vec{F}\right).\vec{n}\,d\,\vec{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **(20)**

2011

- For two vectors $\vec{a}$ and $\vec{b}$ given respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$
  Determine: $(i)\frac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$ and $(ii)\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **(10)**
- If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad\,v$, find the value of $\vec{f}\cdot curl\,\vec{f}$ **(10)**
- Examine whether the vectors $\nabla u, \nabla v$ and ∇w are coplanar, where u, v and w are the scalar functions defined by: $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = yz + zx + xy$. **(15)**
- If $\vec{u} = 4y\hat{i} + x\hat{j} - 2z\hat{k}$, calculate the double integral $\iint\left(\nabla\times\vec{u}\right)\cdot d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$. **(15)**
- If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal. **(15)**
- Verify Gauss Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 1$. **(15)**

2010

- Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at the point (2, 1) in the direction of a unit vector which makes an angle of π/3 with the x – axis. **(12)**
- Show that the vector field defined by the vector function $\vec{V} = xyz(yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k})$ is conservative. **(12)**
- Prove that $div\left(f\vec{V}\right) = f\left(div\vec{V}\right) + \left(grad\,f\right)\vec{V}$ where f is a scalar function. **(20)**
- Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}\,dA$ where $\vec{V} = x^2z\,\vec{i} + y\,\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane z = 4y. **(20)**

- Verify Green's theorem for ; $e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ the path of integration being the boundary of the square whose vertices are (0, 0), (π/2, 0), (π/2, π/2) and (0, π/2). **(20)**

2009

- Show that $div(grad\,r^n) = n(n+1)r^{n-2}$ Where $r = \sqrt{x^2 + y^2 + z^2}$. **(12)**
- Find the directional derivatives of –
  (i) $4xz^3 - 3x^2y^2z^2$ at (2, -1, 2) along z – axis;
  (ii) $x^2yz + 4xz^2$ at (1, -2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(12)**
- Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ under the field of force given by $\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$ **(20)**
- Using divergence theorem, evaluate $\iint_S \vec{A}.d\vec{S}$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(20)**
- Find the value of $\iint_S (\vec{\nabla}\times\vec{F}).\,d\vec{S}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane z = 0, when $\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$. **(20)**

2008

- Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2)
- Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).
- P.T $\nabla^2 f(r) = \frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = (x^2 + y^2 + z^2)^{1/2}$ Hence find f(r) such that $\nabla^2 f(r) = 0$.
- Show that for the space curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$ the curvature and torsion are same at every point.
- Evaluate $\int_c \vec{A}.d\vec{r}$ along the curve $x^2 + y^2 = 1,\ z = 1$ from (0, 1, 1) to (1, 0,1) if $\vec{A} = (yz + 2x)\ \hat{i} + xz\ \hat{j} + (xy + 2z)\hat{k}$.
- Evaluate $\iint_s \vec{F}\cdot\hat{n}\,dS$ where $\vec{F} = 4x\,\hat{i} - 2y^2\,\hat{j} + z^2\hat{k}$ and 'S' is the surface of the cylinder bounded by $x^2 + y^2 = 4$, z = 0 and z = 3.

2007

- If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$ determine grad (r^{-1}) in terms of $\hat{r}$ and r.
- Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$
- For any constant vector $\vec{a}$ show that the vector represented by curl $(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z), measured from the origin.
- If $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ find the value(s) of n in order that $r^n\,\vec{r}$ may be (i) solenoidal or (ii) irrotational
- Determine $\int_C (y\,dx + z\,dy + x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2,\ x + y = 2a$ that starts from the point (2a, 0, 0) and goes at first below the z – plane.

2006

- Find the values of constant a, b, and c so that the directional of the function $f = axy^2 + byz + cz^2x^3$

at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to Z–axis.

- If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} - 7\hat{k},$ determine a vector $\vec{R}$ satisfying the vector equations $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$
- Prove that $r^n \vec{r}$ is an irrotational vector for any value of n , but is solenoidal only if n + 3 = 0 .
- If the unit tangent vector $\vec{t}$ and binormal $\vec{b}$ makes angles θ and φ respectively with a constant unit vector $\vec{a}$, prove that $\frac{\sin\theta}{\sin\phi}.\frac{d\theta}{d\phi} - \frac{k}{\tau}$
- Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane z = 0 and bounded by the lines x=0, y = 0 , x =a and y = a , a > 0.

2005

- Show that the volume of the tetrahedron ABCD is $\frac{1}{6}\left(\overline{AB} \times \overline{AC}\right) \cdot \overline{AD}.$ Hence find the volume of the tetrahedron with vertices (2, 2, 2) , (2, 0, 0) , (0, 2, 0) and (0, 0, 2).
- Prove that the curl of a vector field is independent of the choice of co – ordinates.
- The parametric equation of a circular helix is $\vec{r} = a\cos u\,\hat{i} + a\sin u\,\hat{j} + cu\,\hat{k};$ where 'c' is a constant and 'u' is a parameter.
- Find the unit tangent vector $\hat{t}$ at the point 'u' and the arc length measured from u = 0. Also find $\frac{d\hat{i}}{ds}$, where 'S' is the arc length.
- Show that $curl\left(\hat{k} \times grad\frac{1}{r}\right) + grad\left(\hat{k} \cdot grad\frac{1}{r}\right) = 0$ where r is the distance from the origin and $\hat{k}$ is the unit vector in the direction OZ.
- Find the curvature and the torsion of the space curve $x = a\left(3u - u^3\right), y = 3au^2, \; z = a\left(3u + u^3\right).$
- Evaluate $\oiint_S (x^3 dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy)$ by Gauss divergence theorem, where S is the surface of the cylinde $x^2 + y^2 = a^2$ bounded by z =0 and z = b.

2004

- Show that if $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A} \times \vec{B}$ is solenoidal.
- Show that the Frenet – Serret formula can be written in the form $\frac{d\vec{T}}{ds} = \vec{\omega} \times \vec{T}$, $\frac{d\vec{N}}{ds} = \vec{\omega} \times \vec{N}$ and $\frac{d\vec{B}}{ds} = \vec{\omega} \times \vec{B}$

  Where, $\vec{\omega} = \tau\vec{T} + k\vec{B}$
- Prove the identity $\nabla\nabla\left(\vec{A} \cdot \vec{B}\right) = \left(\vec{B} \cdot \nabla\right)\vec{A} + \left(\vec{A} \cdot \nabla\right)\vec{B} + \vec{B} \times \left(\nabla \times \vec{A}\right) + \vec{A} \times \left(\nabla \times \vec{B}\right)$
- Derive the identity $\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)d\forall = \iint_S (\phi\nabla\psi - \psi\nabla\phi).\hat{n}\,dS$, where V is the volume bounded by the closed surface S.
- Verify Stoke's theorem for $\vec{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2003

- Show that if $\vec{a}', \vec{b}'$ *and* $\vec{c}'$ are the reciprocals of the non – coplanar vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, then any vector $\vec{r}$ may be expressed as $\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')b + (\vec{r}.\vec{c}')c.$
- Prove that the divergence of a vector field is invariant w. r. t co – ordinate transformations.
- Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.

- Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A\times(\nabla\times A)$

  Where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

- Find the radii of curvature and torsion at a point of intersection of the surfaces

  $x^2 - y^2 = c^2,\ y = x\ \tanh\left(\frac{z}{c}\right)$

- Evaluate $\iint_S curl\ \ A.dS$ where S is the open surface

  $x^2 + y^2 - 4x + 4z = 0,\ z \ge 0$ and

  $A = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$

2002

- Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$.

  Show that $\vec{R}\times\frac{d\vec{R}}{dt} = \frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ where $r = |\vec{r}|$.

- Find the curvature K for the space curve $x = a\cos\theta,\ y = a\sin\theta,\ z = a\theta\tan\alpha$

- Show that curl (curl $\vec{v}$) = grad ($div\,\vec{v}$) - $\nabla^2\vec{v}$

- Let D be a closed and bounded region having boundary S. Further let 'f' be a scalar function having second order partial derivatives defined on it. Show that

  $\iint_S (f\ grad\ f).\hat{n}\,dS = \iiint_V \left[|\ grad\ |^2 + f\nabla^2 f\right] d\forall$

  Hence or otherwise evaluate $\iint_S (f\ grad\ f).\hat{n}\,dS$

  for f = 2x + y +2z over $S \equiv x^2 + y^2 + z^2 = 4$

- Find the values of constants a, b, and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at (1, -1, 1) is in the direction parallel to y axis and has magnitude 6.

2001

- Find the length of the arc of the twisted curve $\vec{r} = (3t, 3t^2, 2t^3)$ from the point $t = 0$ to the point $t = 1$. Find also the unit tangent $\vec{t}$, unit normal $\vec{n}$ and the unit binormal $\vec{b}$ at $t = 1$.

- Show that curl $\frac{\vec{a}\times\vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a}.\vec{r})$ where $\vec{a}$ *is a* constant vector.

- Find the directional derivative of $f = x^2yz^3$ along $x = e^{-t},\ y = 1 + 2\sin t,\ z = t - \cos t$ at t = 0.

- Show that the vector field defined by $F = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find also the scalar 'u' such that F = grad u.

- Verify Gauss divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4$, z = 0 & z = 3.

2000

- In what direction from the point (-1 , 1 , 1) is the directional derivative of $f = x^2yz^3$ a maximum? compute its magnitude.

- Show that

  (i) $(A+B).(B+C)\times(C+A) = 2A.B\times C$

  (ii) $\nabla\times(A\times B) = (B.\nabla)A - B(\nabla.A) - (A.\nabla)B + A(\nabla.B)$

(1990)

- Evaluate $\iint_S F.\hat{n}\,dS$ where $F = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the surface of the parallelopiped bounded by $x = 0,\ y = 0,\ z = 0,\ x = 2,\ y = 1$ *and* $z = 3$.

1999

- If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors A ,B, C prove that $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$ is a vector perpendicular to the plane ABC.

- If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find $\nabla\times\vec{F}$.

- Evaluate $\int_c \left(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right)$; (by Green's theorem), where 'C' is the rectangle whose vertices are (0,0) , (π, 0) $(\pi, \pi/2)\ \&\ (0, \pi/2)$.

- If X, Y, Z are the components of a contra variant vector in rectangular cartesian co-ordinates x,y,z in a three dimensional space, show that the components of the vector in cylindrical co-ordinates

r, θ, Z are $X\cos\theta + Y\sin\theta, \frac{-x}{r}\sin\theta + \frac{y}{r}\cos\theta, Z$

1998

❖ If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point P (x , y , z), then find the values of grad $(r_1 \cdot r_2)$ and curl $(r_1 \times r_2)$

❖ Show that $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$ if either $\vec{b} = 0$ (or any other vector is '0') or $\vec{c}$ is collinear with $\vec{a}$ or $\vec{b}$ is orthogonal to $\vec{a}$ and $\vec{c}$ (both).

1997

❖ Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are three given non coplanar vectors, then any vector $\vec{F}$ can be put in the form $\vec{F} = \alpha\vec{B}\times\vec{C} + \beta\vec{C}\times\vec{A} + \gamma\vec{A}\times\vec{B}$. For a given $\vec{F}$ determine α ,β ,γ.

❖ Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, and z = 0 and $z = 3$.

1996

❖ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, show that

(i) $\vec{r} \times grad\, f(r) = 0$

(ii) $div\,(r^n\vec{r}) = (n+3)r^n$

❖ Verify Gauss divergence theorem for $\vec{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$, on the tetrahedron x = y = z = 0, x +y + z=1

1994

❖ If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$.

evaluate $\iint_S (\nabla\times\vec{F}).\hat{n}dS$.

❖ Evaluate $\iint_s \nabla\times\vec{F}.\hat{n}\,ds$, where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$.

1992

❖ If $\vec{f}(x,y,z) = (y^2+z^2)\,\hat{i} + (z^2+x^2)\,\hat{j} + (x^2+y^2)\hat{k}$ then calculate $\int_C \vec{f}.d\vec{x}$ where 'C' consists of

(i) The line segment from (0,0,0) to (1,1,1)

(ii) The three line segments AB,BC and CD, where A,B,C and D are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)

(iii) The curve $\vec{x} = u\hat{i} + u^2\hat{j} + u^3\hat{k}$, u from 0 to 1.

❖ If $\vec{a}$ and $\vec{b}$ are constant vectors , show that

(i) $div\{\vec{x}\times(\vec{a}\times\vec{x})\} = -2\vec{x}.\vec{a}$

(ii) $div\{(\vec{a}\times\vec{x})\times(\vec{b}\times\vec{x})\} = 2\vec{a}.(\vec{b}\times\vec{x}) - 2\vec{b}.(\vec{a}\times\vec{x})$

1991

❖ If φ be a scalar point function and F be a vector point function, show that the components of F normal and tangential to surface $\phi = 0$ at any point there of are $\frac{(F.\nabla\phi)\nabla\phi}{(\nabla\phi)^2}$ and $\frac{\nabla\phi\times(F\times\nabla\phi)}{(\nabla\phi)^2}$

❖ Find the value of $\int$ curl F. dS taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$, for which z ≥ 0,

when $F = (y^2+z^2-x^2)\,\hat{i} + (z^2+x^2-y^2)\,\hat{j} + (x^2+y^2-z^2)\hat{k}$.

1989

❖ Define the curl of a vector point function

❖ Prove that $\nabla\times\left(\frac{\vec{r}}{r^2}\right) = 0$ where $\vec{r} = (x, y, z)$ and $r = |\vec{r}|$.

1988

❖ Define the divergence of a vector point function , prove that $div\,(\vec{u}\times\vec{v}) = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$.

(1986)

- Using Gauss divergence theorem, evaluate $\iint_S (x\hat{i}+y\hat{i}+z^2\hat{k}).\hat{n}\,ds$ where S is the closed surface bounded by the cone $x^2+y^2=2$ and the plane Z=1 and $\hat{n}$ is the outward unit normal to S.

1987

- Show that for a vector field $\vec{f}$, curl (curl $\vec{f}$) = grad (div $\vec{f}$) - $\nabla^2\vec{f}$.

- If $\vec{r}$ is the position vector to a point whose distance from the origin is r , prove that $div\,\vec{f}=0$ if $\vec{f}=\frac{\vec{r}}{r^3}$.

- Prove that for a three vectors $\vec{a},\vec{b},\vec{c}$ $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\vec{b}\left(\vec{a}.\vec{c}\right)-\vec{c}\left(\vec{a}.\vec{b}\right)$ and explain its geometric meaning. ***(1990)***

1986

- Let $\vec{a},\vec{b}$ be given vectors in the three dimensional Euclidean space E_3 and let $\phi(\vec{x})$ be a scalar field of the vectors $\vec{x}$ also of E_3.

  If $\phi(\vec{x})=(\vec{x}\times\vec{a}).(\vec{x}\times\vec{b})$, show that grad $\phi(i.e,\nabla\phi(\vec{x}))=\vec{b}\times(\vec{x}\times\vec{a})+\vec{a}\times(\vec{x}\times\vec{b})$.

- If $\vec{f},\vec{g}$ are two vector fields in E_3 and if 'div', 'curl' are defined on an open set $S\subset E_3$ show that $div(\vec{f}\times\vec{g})=\vec{g}.\,curl\,\vec{f}-\vec{f}.curl\,\vec{g}$. ***(1988)***

1985

- If P,Q,R are points (3,-2,-1) , (1,3,4) , (2,1,-2) respectively. Find the distance from P to the plane OQR, where 'O' is the origin.
- Find the angle between the tangents to the curve $\vec{r}=t^2\hat{i}-2t\,\hat{j}+t^3\hat{k}$ at the points t=1 and t =2
- Find div F and curl F, where $F=\nabla(x^3+y^3+z^3-3xyz)$

1983

- Prove that curl (curl F) = grad div F - $\nabla^2 F$.

❖❖❖

IFoS
PREVIOUS YEARS QUESTIONS (2020-2000) SEGMENT-WISE

VECTOR ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2020

❖ Prove that for a vector $\vec{a}$,

$\nabla(\vec{a}\cdot\vec{r}) = \vec{a}$; where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, r = |\vec{r}|$.

Is there any restriction on $\vec{a}$?

Further, show that

$$\vec{a}\cdot\nabla\left(\vec{b}\cdot\nabla\frac{1}{r}\right) = \frac{3(\vec{a}.\vec{r})(\vec{b}.\vec{r})}{r^5} - \frac{\vec{a}\cdot\vec{b}}{r^3}$$

Give an example to verify the above. **[08]**

❖ A tangent is drawn to a given curve at some point of contact. B is a point on the tangent at a distance 5 units from the point of contact. Show that the curvature of the locus of the point B is

$$\frac{\left[25\kappa^2\tau^2(1+25\kappa^2)+\left(\kappa+5\frac{d\kappa}{ds}+25\kappa^3\right)\right]^{1/2}}{(1+25\kappa^2)^{3/2}}$$

Find the curvature and torsion of the curve $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. **[15]**

❖ Given a portion of a circular disc of radius 7 units and of height 1.5 units such that x, y, z ≥ 0. Verify Gauss Divergence Theorem for the vector field $\vec{f} = (z, x, 3y^2z)$ over the surface of the above mentioned circular disc. **[15]**

❖ Derive expression of ∇f in terms of spherical coordinates.

Prove that $\nabla^2(fg) = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\,\nabla^2 f$ for any two vector point functions f(r, θ, ϕ) and g(r, θ, ϕ). Construct one example in three dimensions to verify this identity. **[10]**

2019

❖ Let $\bar{r} = \bar{r}(s)$ represent a space curve. Find $\frac{d^3\bar{r}}{ds^3}$ in terms of $\bar{T}, \bar{N}$ and $\bar{B}$, where $\bar{T}, \bar{N}$ and $\bar{B}$, represent tangent, principal normal and binormal respectively. Compute $\frac{d\bar{r}}{ds}\cdot\left(\frac{d^2\bar{r}}{ds^2}\times\frac{d^3\bar{r}}{ds^3}\right)$ in terms of radius of curvature and the torsion. **(08)**

❖ Evaluate $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$. **(08)**

❖ Verify Stoke's theorem for $\bar{V} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(10)**

❖ Derive the Frenet-Serret formula. Verify the same for the space curve x = 3 cos t, y = 3 sin t, z = 4t. **(10)**

❖ Derive $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in spherical coordinates and compute $\nabla^2\left(\frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right)$ in spherical coordinates. **(15)**

2018

❖ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $f(r)$ is differentiable, show that $\text{div}[f(r)\vec{r}] = rf'(r) + 3f(r)$. Hence or otherwise show that $div\left(\frac{\vec{r}}{r^3}\right) = 0$. **(08)**

❖ Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from (1, –2, 1) to (3, 1, 4). **(15)**

- Let α be a unit-speed curve in $\mathbf{R}^3$ with constant curvature and zero torsion. Show that α is (part of) a circle. **(10)**
- For a curve lying on a sphere of radius a and such that the torsion is never 0, show that $\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2\tau}\right)^2 = a^2.$ **(10)**

2017

- Prove that
$$\nabla^2 r^n = n(n+1)r^{n-2}$$
and that $r^n\vec{r}$ is irrotational, where $r = |\vec{r}| = \sqrt{x^2+y^2+z^2}.$ **(8)**
- Using Stokes' theorem, evaluate
$$\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz],$$
where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(15)**
- Evaluate
$\iint_S (\nabla \times \vec{f}) \cdot \hat{n}\, dS,$ where S is the surface of the cone, $z = 2 - \sqrt{x^2+y^2}$ above xy-plane and
$\vec{f} = (x-z)\hat{i} + (x^3+yz)\hat{j} - 3xy^2\hat{k}.$ **(10)**
- Find the curvature and torsion of the circular helix
$\vec{r} = a(\cos\theta, \sin\theta, \theta\cot\beta),$
β is the constant angle at which it cuts its generators. **(10)**
- If the tangent to a curve makes a constant angle α, with a fixed line, then prove that k cos $\alpha \pm \tau$ sin $\alpha = 0$.
Conversely, if $\frac{k}{\tau}$ is constant, then show that the tangent makes a constant angle with a fixed direction. **(10)**

2016

- If E be the solid bounded by the xy plane and the paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \overline{F}.dS$ where S is the surface bounding the volume E and $\overline{F} = \left(zx\sin yz + x^3\right)\hat{i} + \cos yz\hat{j} + \left(3zy^2 - e^{\lambda^2+y^2}\right)\hat{k}$. **(08)**
- Evaluate $\iint_S (\nabla \times \overline{f}).\hat{n}dS$ for $\overline{f} = \left(2x - y\right)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. **(10)**
- State Stokes' theorem. Verify the Stokes' theorem for the function $\overline{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane z = x and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. **(15)**
- Prove that $\overline{a}\times\left(\overline{b}\times\overline{c}\right) = \left(\overline{a}\times\overline{b}\right)\times\overline{c}$, if and only if either $\overline{b} = \overline{0}$ or $\overline{c}$ is collinear with $\overline{a}$ or $\overline{b}$ is perpendicular to both $\overline{a}$ and $\overline{c}$. **(10)**

2015

- Find the curvature and torsion of the curve $x = a$ cos$t$, $y = a$ sin t, $z = bt$. **(08)**
- Examine if the vector field defined by $\vec{F} = 2xyz^3 \hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F}$ = grad ϕ. **(10)**
- Using divergence theorem, evaluate
$\iint_S (x^3\, dydz + x^2\, ydzdx + x^2\, zdydx)$
where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **(15)**
- If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S (\nabla \times \vec{F}).\hat{n}dS,$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. **(10)**

2014

- For three vectors show that:
$\overline{a}\times(\overline{b}\times\overline{c}) + \overline{b}\times(\overline{c}\times\overline{a}) + \overline{c}\times(\overline{a}\times\overline{b}) = 0$ **(08)**

- For the vector $\overline{A} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{x^2 + y^2 + z^2}$ examine if $\overline{A}$ is an irrotational vector. Then determine ϕ such that $\overline{A} = \nabla\phi$. **(10)**

- Evaluate $\iint_S \nabla \times \overline{A} \cdot \overline{n}\, dS$ for $\overline{A} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane. **(15)**

- Verify the divergence theorem for $\overline{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. **(15)**

2013

- $\vec{F}$ being a vector, prove that curl curl $\vec{F}$ = grad div $\vec{F} - \nabla^2\vec{F}$ where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$. **(08)**

- Evaluate $\int_S \vec{F}.d\vec{S}$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$. **(13)**

- Verify the Divergence theorem for the vector function $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. **(14)**

2012

- If $u = x + y + z$, $v + x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that grad u, grad v and grad w are coplanar. **(08)**

- Find the value of $\iint_s \left(\vec{\nabla} \times \vec{F}\right).\vec{ds}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane z = 0, when $\vec{F} = \left(y^2 + z^2 - x\right)\vec{i} + \left(z^2 + x^2 - y^2\right)\vec{j} + \left(x^2 + y^2 - z^2\right)\vec{k}$. **(10)**

- Find the value of the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$ where the vector field, $\vec{F} = (\sin y)\,\vec{i} + x(1 + \cos y)\,\vec{j}$. **(10)**

2011

- Verify Green's theorem in the plane to $\oint_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$. Where C is the boundary of the region enclosed by the curves $y = \sqrt{x}$ *and* $y = x^2$. **(10)**

- The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is $\vec{r} = \left(t^2 - 2t\right)\hat{i} + \left(\dfrac{1}{2}t^2 + 1\right)\hat{j} + \dfrac{1}{2}t^2\,\hat{k}$, At time $t = 1$, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. **(10)**

- Find the curvature, torsion and the relation between the arc length S and parameter u for the curve: $\vec{r} = \vec{r}(u) = 2\log_e u\ \hat{i} + 4u\ \hat{j} + (2u^2 + 1)\ \hat{k}$ **(10)**

- Prove the vector identity: $curl\left(\vec{f} \times \vec{g}\right) = \vec{f}\, div\, \vec{g} - \vec{g}\, div\, \vec{f} + \left(\vec{g}.\nabla\right)\vec{f} - \left(\vec{f}.\nabla\right)\vec{g}$ and verify it for the vectors $\vec{f} = x\,\hat{i} + z\,\hat{j} + y\,\hat{k}$ and $\vec{g} = y\,\hat{i} + z\hat{k}$. **(10)**

- Evaluate the line integral $\oint_C (\sin x\, dx + y^2 dy - dz)$, where C is the circle $x^2 + y^2 = 16, z = 3$, by using Stokes' theorem. **(10)**

2010

- Find the directional derivation of $\overrightarrow{V^2}$, Where, $\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point $(2, 0, 3)$ in the direction of the outward normal to the surface $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1) **(08)**

❖ **(1)** Show that $\vec{F} = \left(2xy + z^2\right)\vec{i} + x^2\vec{j} + 3z^2 x\vec{k}$ is a conservative field. Find its scalar potnetial and also the work done in moving a particle from (1, – 2, 1) to (3, 1, 4).

(2) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right) f'(r) + f''(r)$,

Where $r = \sqrt{x^2 + y^2 + z^2}$. **(10)**

❖ Use divergence theorem to evaluate, $\iint_s \left(x^3\,dy\,dz + x^2 y\,dz\,dx + x^2 z\,dy\,dx\right)$, Where S is the sphere $x^2 + y^2 + z^2 = 1$. **(10)**

❖ If $\vec{A} = 2y\,\vec{i} - z\,\vec{j} - x^2\,\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$,evaluate the surface integral, $\iint_S \vec{A}.\,\hat{n}\,\overrightarrow{dS}$. **(10)**

❖ Use Green's theorem in a plane to evaluate the integral, $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the surface in the xy - plane enclosed by, y = 0 and the semi-circle, $y = \sqrt{1 - x^2}$. **(10)**

2009

❖ Verify Green's theorem in the plane for $\oint_C \left[\left(xy + y^2\right)dx + x^2 dx\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(10)**

❖ Show that

$\bar{A} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$

is irrotational. Find a scalar function ϕ such that $\bar{A} = grad\ \ \phi$. **(10)**

❖ Let $\psi(x, y, z)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. **(08)**

❖ Verify stokes theorem for

$\bar{A} = (y - z + 2)\hat{i} + \left(yz + 4\right)\hat{j} - xz\hat{k}$

Where S is the surface of the cube $x = 0$, $y = 0$ $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. **(12)**

❖ Show that, if $\bar{r} = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$ is a space curve, $\frac{d\bar{r}}{ds}.\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3} = \frac{\tau}{\rho^2}$, where τ is the torsion and ρ the radius of curvature **(10)**

2008

❖ Show that $\oint_S \frac{ds}{\sqrt{a^2x^2 + b^2y^2 + c^2z^2}} = \frac{4\pi}{\sqrt{abc}}$,

Where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ **(10)**

❖ Find the unit vector along the normal to the surface $z = x^2 + y^2$ at the point $(-1, -2, 5)$. **(10)**

❖ Prove that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is $\vec{V}\frac{d\vec{v}}{dt} = 0$. **(10)**

❖ If $\vec{F} = 2x^2\,\hat{i} - 4yz\,\hat{j} + zx\,\hat{k}$, , evaluate $\iint_S \vec{F}.\vec{n}\,ds$

Where S is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

2007

❖ Evaluate $\int_C \vec{F}.d\vec{r}$ Where

$\vec{F} = C\left[-3a\sin^2\theta\cos\theta\vec{i} + a\left(2\sin\theta - 3\sin^2\theta\right)\vec{j} + b\sin 2\theta\vec{k}\right]$

and the curve C is given by $\vec{r} = a\cos\theta\vec{i} + a\sin\theta\vec{j} + b\theta\vec{k}$ θ varying from $\pi/4\ to\ \pi/2$. **(10)**

❖ Show that $curl\left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^3}\left(\vec{a}.\vec{r}\right)$ Where $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ **(10)**

❖ Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$. **(10)**

❖ Evaluate the surface integral $\int_S \left(yz\vec{i} + zx\vec{j} + xy\vec{k} \right) d\vec{a}$, Where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first otant. **(10)**

❖ Apply stokes theorem to Prove that $\int_c \left(ydx + zdy + xdz \right) = -2\sqrt{2}\pi a^2$,
Where C is the curve given by
$x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$. **(10)**

2006

❖ If $\vec{f} = 3xy\hat{i} - y^2\hat{j}$, determine the value of $\int_C \vec{f}.dr$, Where C is the curve y = $2x^2$in the xy-plane from (0, 0) to (1, 2). **(10)**

❖ If $u\vec{f} = \vec{\nabla} V$ Where *u*, *v* are scalar fields and $\vec{f}$ is a vector field , find the value of $\vec{f}.curl\vec{f}$. **(10)**

❖ If O be the origin, *A*, *B* two fixed points and *P*(*x*, *y*, *z*) a variables point, show that $curl\left(\overrightarrow{PA} \times \overrightarrow{PB}\right) = 2\left(\overrightarrow{AB}\right)$. **(10)**

❖ Using stokes theorem, determine the value of the integral $\int_\Gamma \left(y\ dx + z\ dy + xdz \right)$, Where Γ is the curve defined by $x^2 + y^2 + z^2 = a^2$, $x + z = a$ **(10)**

❖ Prove that the cylinderical coordinate system is orthogonal **(10)**

2005

❖ For the curve $\vec{r} = a\left(3t - t^3\right)\vec{i} + 3at^2\vec{j} + a\left(3t + t^3\right)\vec{k}$, a being a constant. Show that the redius of curvature is equal to its radius of torsion **(10)**

❖ Find *f*(*r*) if $f(r)\vec{r}$ is both solenoidal and irrotational. **(10)**

❖ Evaluate $\iint_S \vec{F}.d\vec{s}$ Where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and '*S*' is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant. **(10)**

❖ Verify the divergence theorem for $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(10)**

❖ By using vector methods, find an equation for the tangent plane to the surface $z = x^2 + y^2$ at the point (1, – 1, 2). **(10)**

2004

❖ Evaluate $\int_C \vec{F}.d\vec{r}$ for the field $\vec{F} = grad\left(xy^2z^3\right)$
Where C is the ellipse in which the plane $z = 2x + 3y$ cuts the cylinder $x^2 + y^2 = 12$ counter clockwise as viewed from the positive end of the *z-axis* looking towards the origin. **(10)**

❖ Show that $div\left(\vec{A} \times \vec{B}\right) = \vec{B}\ curl\ \vec{A} - \vec{A}\ .curl\ \vec{B}$ **(10)**

❖ Evaluate $Curl\left[\dfrac{\left(2\vec{i} - \vec{j} + 3\vec{k}\right) \times \vec{r}}{r^n}\right]$
Where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r^2 = x^2 + y^2 + z^2$. **(10)**

❖ Evaluate $\iint_S \left(x\vec{i} + y\vec{j} + z\vec{k} \right) \vec{n}\ ds$. Where S is the surface $x + y + z = 1$ lying in the first octant. **(10)**

❖ Expess $\nabla^2 u$ in spherical polar coordinates. **(10)**

2003

❖ Find the expression for curvature and torsion at a point on the curve
$x = a\cos\theta, y = a\sin\theta, z = a\theta\cot\beta$. **(10)**

❖ If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$. Find *f*(*r*) such that $\nabla^2 f(r) = 0$ **(10)**

❖ If $\vec{F}$ is solenoidal , Prove that
$curl\ curl\ curl\ curl\ \vec{F} = \nabla^4\vec{F}$ **(10)**

❖ Verify stoke's Theorem when $\vec{F} = (2xy - x^2)\vec{i} - (x^2 - y^2)\vec{d}$ & C is the boundary of the region closed by the parabolas $y^2 = x$ and $x^2 = y$. **(10)**

❖ Express $\nabla \times \vec{F}$ and $\nabla^2 \phi$ in cylinderical coordinates. **(10)**

2002

❖ Find the curvature and torsion of the curve, $x = \frac{2t+1}{t-1}$, $y = \frac{t^2}{t-1}$, $z = t+2$. Interpret your answer. **(10)**

❖ State stoke's theorem and then verify if for $\vec{A} = (x^2+1)\vec{i} + xy\,\hat{j}$ integrated round the square in the plane $z = 0$ whose sides are along the lines. $x = 0, y = 0, x = 1, y = 1$. **(10)**

❖ Prove that

$(i)\ \vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \times \nabla)\vec{A} - \vec{B}(\vec{\nabla} \times \vec{A}) - (\vec{A} \times \vec{\nabla})\vec{B}$

$(ii)\ curl \frac{\vec{a} \times \vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^3}(\vec{a}.\vec{r}),$

$\vec{a} = cons\tan t\, vector.$ **(10)**

❖ Show that if $A \neq \vec{0}$ and both of the conditions $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ hold simultaneou-sly then $\vec{B} = \vec{C}$ but if only one of these conditions holds then $\vec{B} \neq \vec{C}$ necessarily. **(10)**

❖ Prove the following

(i) *If* u_1, u_2, u_3 are general coordinates, then $\frac{\partial \vec{r}}{\partial u_1} \times \frac{\partial \vec{r}}{\partial u_2} \times \frac{\partial \vec{r}}{\partial u_3}$ *and* $\vec{\nabla} u_1, \vec{\nabla} u_2, \vec{\nabla} u_3$ are reciprocal system of vectors.

$(ii)\left(\frac{\partial \vec{r}}{u_1} . \frac{\partial \vec{r}}{u_2} \times \frac{\partial \vec{r}}{u_3}\right)\left(\vec{\nabla} u_1 . \vec{\nabla} u_2 \times \vec{\nabla} u_3\right) = 1$ **(10)**

2001

❖ Find an equation for the plane passing through the points $P_1(3,1,-2)$, $P_2(-1,2,4)$, $P_3(2,-1,1)$ by using vector method. **(10)**

❖ Prove that $\nabla \times (\nabla \times \overline{A}) = -\nabla^2 \overline{A} + \nabla(\nabla.\overline{A})$ **(10)**

❖ If $\nabla.\overline{E}, \nabla.\overline{H}, \nabla \times \overline{E} = \frac{\partial \overline{H}}{\partial t}, \nabla \times \overline{H} = \frac{\partial \overline{E}}{\partial t}$ Show that $\overline{E}$ & $\overline{H}$ satisfy $\nabla^2 u = -\frac{\partial^2 \overline{u}}{\partial t^2}$ **(10)**

❖ Given the space Curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. Find (1) the curvature ρ (2) the torsion τ. **(10)**

❖ If $F = (y^2 + z^2 - x^2)i + (z^2 + x^2 - y^2)j + (x^2 + y^2 - z^2)k,$ evaluate $\iint_s curl\,\overline{F}.\hat{n}\,ds,$ taken over the portion of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ above the plane z = 0 and verify stokes theorem. **(10)**

2000

❖ Prove the identities:

(1) Curl grad $\phi = 0$, (2) div curl $f = 0$

If $\overrightarrow{OA} = ai$, $\overrightarrow{OB} = aj$, $\overrightarrow{OC} = ak$ form three coterminous edges of a cube and s denotes the surface of the cube, evaluate $\int_s \{(x^3 - yz)i - 2x^2yj + 2k\}.nds$ by expressing it as volume integral, Where n is the unit outward normal to ds. **(20)**

❖❖❖