(d) A hollow conical vessel floats in water with its vertex downwards and a certain depth of its axis immersed. When water is poured into it up to the level originally immersed, it sinks till its mouth is on a level with the surface of the water. What portion of axis was originally immersed ? 10

8. (a) Show that

$$\bar{A} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k},$$

is irrotational. Find a scalar function ϕ such that $\bar{A} = \text{grad}\ \phi$. 10

(b) Let $\psi(x, y, z)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. 8

(c) Verify Stokes' theorem for

$$\bar{A} = (xy - z + 2)\,\hat{i} + (yz + 4)\,\hat{j} - xz\hat{k},$$

where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. 12

(d) Show that, if $\bar{r} = x(s)\,\hat{i} + y(s)\,\hat{j} + z(s)\,\hat{k}$ is a space curve, $\dfrac{d\bar{r}}{ds} \cdot \dfrac{d^2\bar{r}}{ds^2} \times \dfrac{d^3\bar{r}}{ds^3} = \dfrac{\tau}{\rho^2}$, where τ is the torsion and ρ the radius of curvature. 10
