Q. 8. (a) If $\bar{A} = 2\bar{i} + \bar{k}$, $\bar{B} = \bar{i} + \bar{j} + \bar{k}$, $\bar{C} = 4\bar{i} - 3\bar{j} - 7\bar{k}$, determine a vector $\bar{R}$ satisfying the vector equations.

$$\bar{R} \times \bar{B} = \bar{C} \times \bar{B} \text{ and } \bar{R} \cdot \bar{A} = 0$$ 15

(b) Prove that $r^n \bar{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n + 3 = 0$. 15

(c) If the unit tangent vector $\bar{t}$ and binormal $\bar{b}$ make angles θ and ϕ respectively with a constant unit vector $\bar{a}$, prove that

$$\frac{\sin\theta}{\sin\phi} \cdot \frac{d\theta}{d\phi} = -\frac{k}{\tau}$$ 15

(d) Verify Stokes' theorem for the function

$$\bar{F} = x^2\hat{i} - xy\hat{j}$$

integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = a$, $a > 0$. 15