IFoS
PREVIOUS YEARS QUESTIONS (2017-2000) SEGMENT-WISE

VECTOR ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2017

- Prove that

  $\nabla^2 r^n = n(n+1)r^{n-2}$

  and that $r^n\vec{r}$ is is irrotational, where $r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$. **(8)**

- Using Stokes' theorem, evaluate

  $\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz]$,

  where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(15)**

- Evaluate

  $\iint_S (\nabla \times \vec{f}) \cdot \hat{n}\, dS$,

  where S is the surface of the cone, $z = 2 - \sqrt{x^2+y^2}$ above xy-plane and

  $\vec{f} = (x-z)\hat{i} + (x^3+yz)\hat{j} - 3xy^2\hat{k}$. **(10)**

- Find the curvature and torsion of the circular helix $\vec{r} = a(\cos\theta, \sin\theta, \theta\cot\beta)$, β is the constant angle at which it cuts its generators. **(10)**

- If the tangent to a curve makes a constant angle α, with a fixed line, then prove that $k\cos\alpha \pm \tau\sin\alpha = 0$. Conversely, if $\frac{k}{\tau}$ is constant, then show that the tangent makes a constant angle with a fixed direction. **(10)**

2016

- If E be the solid bounded by the xy plane and the paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \overline{F}.dS$ where S is the surface bounding the volume E and $\overline{F} = (zx\sin yz + x^3)\hat{i} + \cos yz\hat{j} + (3zy^2 - e^{\lambda^2+y^2})\hat{k}$. **(8)**

- Evaluate $\iint_S (\nabla \times \overline{f}).\hat{n}dS$ for $\overline{f} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. **(10)**

- State Stokes' theorem. Verify the Stokes' theorem for the function $\overline{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane z = x and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. **(15)**

- Prove that $\overline{a} \times (\overline{b} \times \overline{c}) = (\overline{a} \times \overline{b}) \times \overline{c}$, if and only if either $\overline{b} = \overline{0}$ or $\overline{c}$ is collinear with $\overline{a}$ or $\overline{b}$ is perpendicular to both $\overline{a}$ and $\overline{c}$. **(10)**

2015

- Find the curvature and torsion of the curve $x = a\cos t$, $y = a\sin t$, $z = bt$. **(8)**

- Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F} = \text{grad}\,\phi$. **(10)**

- Using divergence theorem, evaluate

  $\iint_S (x^3\,dydz + x^2y\,dzdx + x^2z\,dydx)$

  where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **(15)**

- If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate

$\iint_S (\nabla\times\vec{F}).\hat{n}dS,$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. **(10)**

2014

❖ For three vectors show that:

$\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})+\bar{c}\times(\bar{a}\times\bar{b})=0$ **(8)**

❖ For the vector $\bar{A}=\dfrac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}$ examine if $\bar{A}$ is an irrotational vector. Then determine ϕ such that $\bar{A}=\nabla\phi.$ **(10)**

❖ Evaluate $\iint_S \nabla\times\bar{A}\cdot\bar{n}\,dS$ for

$\bar{A}=(x^2+y-4)\hat{i}+3xy\hat{j}+(2xz+z^2)\hat{k}$ and S is the surface of hemisphere $x^2+y^2+z^2=16$ above xy plane. **(15)**

❖ Verify the divergence theorem for

$\bar{A}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ over the region

$x^2+y^2=4,\ z=0,\ z=3.$ **(15)**

2013

❖ $\vec{F}$ being a vector, prove that

curl curl $\vec{F}$ = grad div $\vec{F}-\nabla^2\vec{F}$

where $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}.$ **(8)**

❖ Evaluate $\int_S \vec{F}.d\vec{S},$

where $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$

and S is the surface bounding the region

$x^2+y^2=4, z=0$ and $z=3.$ **(13)**

❖ Verify the Divergence theorem for the vector function

$\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$

taken over the rectangular parallelopiped

$0\le x\le a, 0\le y\le b, 0\le z\le c.$ **(14)**

2012

❖ If $u=x+y+z,\quad v=x^2+y^2+z^2,\ w=yz+zx+xy,$ prove that grad u, grad v and grad w are coplanar. **(8)**

❖ Find the value of $\iint_s\left(\vec{\nabla}\times\vec{F}\right).\vec{ds}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane z = 0, when

$\vec{F}=\left(y^2+z^2-x\right)\vec{i}+\left(z^2+x^2-y^2\right)\vec{j}+\left(x^2+y^2-z^2\right)\vec{k}.$

(10)

❖ Find the value of the line integral over a circular path given by $x^2+y^2=a^2$, z = 0 where the vector field, $\vec{F}=(\sin y)\,\vec{i}+x(1+\cos y)\,\vec{j}.$ **(10)**

2011

❖ Verify Green's theorem in the plane to $\oint_C\left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right].$

Where C is the boundary of the region enclosed by the curves $y=\sqrt{x}$ *and* $y=x^2.$ **(10)**

❖ The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is

$\vec{r}=\left(t^2-2t\right)\hat{i}+\left(\frac{1}{2}t^2+1\right)\hat{j}+\frac{1}{2}t^2\hat{k},$

At time $t=1$, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. **(10)**

❖ Find the curvature, torsion and the relation between the arc length S and parameter u for the curve:

$\vec{r}=\vec{r}(u)=2\log_e u\,\hat{i}+4u\,\hat{j}+(2u^2+1)\hat{k}$ **(10)**

❖ Prove the vector identity:

$curl\left(\vec{f}\times\vec{g}\right)=\vec{f}\,div\,\vec{g}-\vec{g}\,div\,\vec{f}+\left(\vec{g}.\nabla\right)\vec{f}-\left(\vec{f}.\nabla\right)\vec{g}$

and verify it for the vectors $\vec{f}=x\,\hat{i}+z\,\hat{j}+y\,\hat{k}$ and $\vec{g}=y\,\hat{i}+z\hat{k}.$ **(10)**

❖ Evaluate the line integral $\oint_C (\sin x\,dx + y^2 dy - dz)$, where C is the circle $x^2 + y^2 = 16, z = 3,$ by using Stokes' theorem. **(10)**

2010

❖ Find the directional derivation of $\overrightarrow{V^2}$, Where, $\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point $(2,0,3)$ in the direction of the outward normal to the surface $x^2 + y^2 + z^2 = 14$ at the point $(3,2,1)$ **(8)**

❖ **(1)** Show that $\vec{F} = (2xy + z^2)\vec{i} + x^2\vec{j} + 3z^2x\vec{k}$ is a conservative field. Find its scalar potnetial and also the work done in moving a particle from $(1,-2,1) to (3,1,4)$.

(2) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right) f'(r) + f''(r)$, Where $r = \sqrt{x^2 + y^2 + z^2}$. (10)

❖ Use divergence theorem to evaluate, $\iint_s (x^3\,dy\,dz + x^2y\,dz\,dx + x^2z\,dy\,dx)$, Where S is the sphere $x^2 + y^2 + z^2 = 1$. **(10)**

❖ If $\vec{A} = 2y\,\vec{i} - z\,\vec{j} - x^2\,\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, evaluate the surface integral, $\iint_S \vec{A}.\hat{n}\,\overrightarrow{dS}$. **(10)**

❖ Use Green's theorem in a plane to evaluate the integral, $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the surface in the xy - plane enclosed by, y = 0 and the semi-circle, $y = \sqrt{1 - x^2}$ **(10)**

2009

❖ Verify Green's theorem in the plane for $\oint_C \left[(xy + y^2)dx + x^2dx\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(10)**

❖ Show that $\bar{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function ϕ such that $\bar{A} = grad\ \ \phi.$ **(10)**

❖ Let $\psi(x, y, z)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. **(8)**

❖ Verify stokes theorem for $\bar{A} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ Where S is the surface of the cube $x = 0$, $y = 0$ $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. **(12)**

❖ Show that, if $\bar{r} = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$ is a space curve, $\frac{d\bar{r}}{ds}.\frac{d^2\bar{r}}{ds^2}\times\frac{d^3\bar{r}}{ds^3} = \frac{\tau}{\rho^2}$, where τ is the torsion and ρ the radius of curvature **(10)**

2008

❖ Show that $\oint_S \frac{ds}{\sqrt{a^2x^2 + b^2y^2 + c^2z^2}} = \frac{4\pi}{\sqrt{abc}}$, Where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ **(10)**

❖ Find the unit vector along the normal to the surface $z = x^2 + y^2$ at the point $(-1,-2,5)$. **(10)**

❖ Prove that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is $\vec{V}\frac{d\vec{v}}{dt} = 0$. **(10)**

❖ If $\vec{F} = 2x^2\,\hat{i} - 4yz\,\hat{j} + zx\,\hat{k}$, , evaluate $\iint_S \vec{F}.\vec{n}\,ds$ Where S is the surface of the cube boundded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

2007

- Evaluate $\int_C \vec{F}.d\vec{r}$ Where $\vec{F}=C\left[-3a\sin^2\theta\cos\theta\vec{i}+a\left(2\sin\theta-3\sin^2\theta\right)\vec{j}+b\sin2\theta\vec{k}\right]$ and the curve C is given by $\vec{r}=a\cos\theta\vec{i}+a\sin\theta\vec{j}+b\theta\vec{k}$ θ varying from $\pi/4$ to $\pi/2$. **(10)**
- Show that $curl\left(\frac{\vec{a}\times\vec{r}}{r^3}\right)=-\frac{\vec{a}}{r^3}+\frac{3\vec{r}}{r^3}\left(\vec{a}.\vec{r}\right)$ Where $\vec{a}$ is a constant vector and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ **(10)**
- Find the curvature and torsion at any point of the curve $x=a\cos2t$, $y=a\sin2t$, $z=2a\sin t$. **(10)**
- Evaluate the surface integral $\int_S\left(yz\vec{i}+zx\vec{j}+xy\vec{k}\right)d\vec{a}$, Where S is the surface of the sphere $x^2+y^2+z^2=1$ in the first otant. **(10)**
- Apply stokes theorem to Prove that $\int_c(ydx+zdy+xdz)=-2\sqrt{2}\pi a^2$, Where C is the curve given by $x^2+y^2+z^2-2ax-2ay=0, x+y=2a$. **(10)**

2006

- If $\vec{f}=3xy\hat{i}-y^2\hat{j}$, determine the value of $\int_C\vec{f}.dr$, Where C is the curve $y=2x^2$ in the xy-plane from (0, 0) to (1, 2). **(10)**
- If $u\vec{f}=\vec{\nabla}V$ Where u,v are scalar fields and $\vec{f}$ is a vector field, find the value of $\vec{f}.curl\vec{f}$. **(10)**
- If O be the origin, A,B two fixed points and $P(x,y,z)$ a variables point, show that $curl\left(\overrightarrow{PA}\times\overrightarrow{PB}\right)=2\left(\overrightarrow{AB}\right)$. **(10)**
- Using stokes theorem, determine the value of the integral $\int_\Gamma(y\,dx+z\,dy+xdz)$, Where Γ is the curve defined by $x^2+y^2+z^2=a^2$, $x+z=a$ **(10)**
- Prove that the cylinderical coordinate system is orthogonal **(10)**

2005

- For the curve $\vec{r}=a\left(3t-t^3\right)\vec{i}+3at^2\vec{j}+a\left(3t+t^3\right)\vec{k}$, a being a constant. Show that the redius of curvature is equal to its radius of torsion **(10)**
- Find $f(r)$ if $f(r)\vec{r}$ is both solenoidal and irrotational. **(10)**
- Evaluate $\iint_S\vec{F}.d\vec{s}$ Where $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$ and 'S' is the part of the sphere $x^2+y^2+z^2=1$ that lies in the first octant. **(10)**
- Verify the divergence theorem for $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$. **(10)**
- By using vector methods, find an equation for the tangent plane to the surface $z=x^2+y^2$ at the point $(1,-1,2)$. **(10)**

2004

- Evaluate $\int_C\vec{F}.d\vec{r}$ for the field $\vec{F}=grad\left(xy^2z^3\right)$ Where C is the ellipse in which the plane $z=2x+3y$ cuts the cylinder $x^2+y^2=12$ counter clockwise as viewed from the positive end of the $z-axis$ looking towards the origin. **(10)**
- Show that $div\left(\vec{A}\times\vec{B}\right)=\vec{B}\ curl\ \vec{A}-\vec{A}\ .curl\ \vec{B}$ **(10)**
- Evaluate $Curl\left[\frac{\left(2\vec{i}-\vec{j}+3\vec{k}\right)\times\vec{r}}{r^n}\right]$

Where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r^2 = x^2 + y^2 + z^2$ **(10)**

❖ Evaluate $\iint_S \left(x\vec{i} + y\vec{j} + z\vec{k}\right)\vec{n}\, ds$. Where S is the surface $x + y + z = 1$ lying in the first octant.**(10)**

❖ Expess $\nabla^2 u$ in spherical polar coordinates. **(10)**

2003

❖ Find the expression for curvature and torsion at a point on the curve $x = a\cos\theta, y = a\sin\theta, z = a\theta\cot\beta.$ **(10)**

❖ If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. Find $f(r)$ such that $\nabla^2 f(r) = 0$ **(10)**

❖ If $\vec{F}$ is solenoidal , Prove that

$curl\ curl\ curl\ curl\ \vec{F} = \nabla^4 \vec{F}$ **(10)**

❖ Verify stoke's Theorem when

$\vec{F} = \left(2xy - x^2\right)\vec{i} - \left(x^2 - y^2\right)\vec{d}$ & C is the boundary of the region closed by the parabolas $y^2 = x$ and $x^2 = y.$ **(10)**

❖ Express $\nabla \times \vec{F}$ and $\nabla^2 \phi$ in cylinderical coordinates. **(10)**

2002

❖ Find the curvature and torsion of the curve, $x = \frac{2t+1}{t-1}$, $y = \frac{t^2}{t-1}$, $z = t + 2$. Interpret your answer. **(10)**

❖ State stoke's theorem and then verify if for $\vec{A} = \left(x^2 + 1\right)\vec{i} + xy\ \hat{j}$ integrated round the square in the plane $z = 0$ whose sides are along the lines. $x = 0, y = 0,\ x = 1, y = 1.$ **(10)**

❖ Prove that

$(i)\ \vec{\nabla} \times \left(\vec{A} \times \vec{B}\right) = \left(\vec{B} \times \nabla\right)\vec{A} - \vec{B}\left(\vec{\nabla} \times \vec{A}\right) - \left(\vec{A} \times \vec{\nabla}\right)\vec{B}$

$(ii)\ curl \frac{\vec{a} \times \vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^3}\left(\vec{a}.\vec{a}\right) = constant\ vector.$ **(10)**

❖ Show that if $A \neq \vec{0}$ and both of the conditions $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ hold simultaneously then $\vec{B} = \vec{C}$ but if only one of these conditions holds then $\vec{B} \neq \vec{C}$ necessarily. **(10)**

❖ Prove the following

(i) If u_1, u_2, u_3 are general coordinates, then $\frac{\partial \vec{r}}{\partial u_1} \times \frac{\partial \vec{r}}{\partial u_2} \times \frac{\partial \vec{r}}{\partial u_3}$ and $\vec{\nabla} u_1, \vec{\nabla} u_2, \vec{\nabla} u_3$ are reciprocal system of vectors.

$(ii) \left(\frac{\partial \vec{r}}{u_1} . \frac{\partial \vec{r}}{u_2} \times \frac{\partial \vec{r}}{u_3}\right)\left(\vec{\nabla} u_1 . \vec{\nabla} u_2 \times \vec{\nabla} u_3\right) = 1$ **(10)**

2001

❖ Find an equation for the plane passing through the points $P_1(3,1,-2)$, $P_2(-1,2,4)$, $P_3(2,-1,1)$ by using vector method. **(10)**

❖ Prove that $\nabla \times \left(\nabla \times \overline{A}\right) = -\nabla^2 \overline{A} + \nabla\left(\nabla . \overline{A}\right)$ **(10)**

❖ If $\nabla . \overline{E}, \nabla . \overline{H}, \nabla \times \overline{E} = \frac{\partial \overline{H}}{\partial t}, \nabla \times \overline{H} = \frac{\partial \overline{E}}{\partial t}$ Show that $\overline{E}$ & $\overline{H}$ satisfy $\nabla^2 u = -\frac{\partial^2 \overline{u}}{\partial t^2}$ **(10)**

❖ Given the space Curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. Find (1) the curvature ρ (2) the torsion τ. **(10)**

❖ If $F = \left(y^2 + z^2 - x^2\right)i + \left(z^2 + x^2 - y^2\right)j + \left(x^2 + y^2 - z^2\right)k,$ evaluate $\iint_s curl\, \vec{F}.\hat{n}\, ds,$ taken over the portion of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ above the plane $z = 0$ and verify stokes theorem. **(10)**

2000

- Prove the identities:

  (1) Curl grad $\phi = 0$, (2) $div\ curl\ f = 0$.

  If $\overrightarrow{OA} = ai$, $\overrightarrow{OB} = aj$, $\overrightarrow{OC} = ak$ form three coterminous edges of a cube and s denotes the surface of the cube, evaluate $\iint_s \left\{ \left(x^3 - yz\right) i - 2x^2 yj + 2k \right\} . nds$ by expressing it as volume integral, Where n is the unit outward normal to ds. **(20)**