IAS
PREVIOUS YEARS QUESTIONS (2017-1983)
SEGMENT-WISE

VECTOR ANALYSIS

2017

- For what values of the constants a, b and c the vector $\bar{V}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(-x+cy+2z)\hat{k}$ is irrotational. find the divergence in cylindrical coordinates of this vector with these values. **(10)**
- The position vector of a moving point at time t is $\vec{r}=\sin t\,\hat{i}+\cos 2t\,\hat{j}+(t^2+2t)\hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time t = 0. **(10)**
- Find the curvature vector and its magnitude at any point $\bar{r}=(\theta)$ of the curve $\bar{r}=(a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **(16)**
- Evaluate the integral : $\iint_S \bar{F}\cdot\hat{n}\,ds$ where $\bar{F}=3xy^2\hat{i}+\left(yx^2-y^3\right)\hat{j}+3zx^2\hat{k}$ and S is a surface of the cylinder $y^2+z^2\le 4, -3\le x\le 3$, using divergence theorem. **(09)**
- Using Green's theorem, evaluate the $\int_C F(\bar{r})\cdot d\bar{r}$ counterclockwise where $F(\bar{r})=\left(x^2+y^2\right)\hat{i}+\left(x^2-y^2\right)\hat{j}$ and $d\bar{r}=dx\hat{i}+dy\,\hat{j}$ and the curve C is the boundary of the region $R=\left\{(x,y)\,|\,1\le y\le 2-x^2\right\}$. **(08)**

2016

- Prove that the vectors $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$, $\vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. **(10)**
- Find f(r) such that $\nabla f=\dfrac{\vec{r}}{r^5}$ and f(1) = 0. **(10)**
- Prove that
$$\oint_C f\,d\vec{r}=\iint_S d\vec{S}\times\nabla f$$ **(10)**
- For the cardioid r = a (1 + cosθ), show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta=0,\dfrac{\pi}{4},\dfrac{\pi}{2}$. **(15)**

2015

- Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z=x^2+y^2-3$ at (2, –1, 2). **(10)**
- Find the value of λ and μ so that the surfaces $\lambda x^2-\mu yz=(\lambda+2)x$ and $4x^2y+z^3=4$ may intersect orthogonally at (1, –1, 2). **(12)**
- A vector field is given by
$$\vec{F}=(x^2+xy^2)\hat{i}+(y^2+x^2y)\hat{j}$$
Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. **(12)**
- Evaluate $\int_C e^{-x}(\sin y\,dx+\cos y\,dy)$, where C is the rectangle with vertices (0, 0), (π, 0), $\left(\pi,\dfrac{\pi}{2}\right),\left(0,\dfrac{\pi}{2}\right)$. **(12)**

2014

- Find the curvature vector at any point of the curve $\bar{r}(t)=t\cos t\,\hat{i}+t\sin t\,\hat{j}, 0\le t\le 2\pi$. Give its magnitude also. **(10)**
- Evaluate by Stokes' theorem

$$\int_{\Gamma}(y\,dx+z\,dy+x\,dz)$$

where Γ is the curve given by $x^2+y^2+z^2-2ax-2ay=0,\ x+y=2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. **(20)**

2013

- Show that the curve

$$\vec{x}(t)=t\hat{i}+\left(\frac{1+t}{t}\right)\hat{j}+\left(\frac{1-t^2}{t}\right)\hat{k}$$

lies in a plane. **(10)**
- Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplace operator. **(10)**
- A curve in space is defind by the vector equation $\vec{r}=t^2\hat{i}+2t\hat{j}-t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t=+1$ and $t=-1$. **(10)**
- By using Divergence Theorem of Gauss, evaluate the surface integral

$$\iint\left(a^2x^2+b^2y^2+c^2z^2\right)^{-1/2}dS,$$

where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$, a, b and c being all positive constants. **(15)**
- Use Stokes theorem to evaluate the line integral $\int_C\left(-y^3dx+x^3dy-z^3dz\right)$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. **(15)**

2012

- If $\vec{A}=x^2yz\vec{i}-2xz^3\vec{j}+xz^2\vec{k}$

$\vec{B}=2z\vec{i}+y\vec{j}-x^2\vec{k}$

Find the value of $\frac{\partial^2}{\partial x\partial y}\left(\vec{A}\times\vec{B}\right)$ at $(1, 0, -2)$. **(12)**
- Derive the Frenet-Serret formulae.

Define the curvature and torsion for a space curve. Compute them for the space curve $x=t,\ y=t^2,\ z=\frac{2}{3}t^3$

Show that the curvature and torsion are equal for this curve. **(20)**
- Verify Green's theorem in the plane for

$$\oint_C\left\{\left\{xy+y^2\right\}dx+x^2dy\right\}$$

where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. **(20)**
- If $\vec{F}=y\vec{i}+(x-2xz)\vec{j}-xy\vec{k}$, evaluate

$$\iint_S\left(\vec{\nabla}\times\vec{F}\right).\vec{n}\,d\vec{s}$$

where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. **(20)**

2011

- For two vectors $\vec{a}$ and $\vec{b}$ given respectively by

$\vec{a}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and $\vec{b}=\sin t\hat{i}-\cos t\hat{j}$

Determine: $(i)\frac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$ and $(ii)\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **(10)**
- If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f}=grad\,v$, find the value of $\vec{f}\cdot curl\,\vec{f}$ **(10)**
- Examine whether the vectors $\nabla u, \nabla v$ and ∇w are coplanar, where u, v and w are the scalar functions defined by: $u=x+y+z$, $v=x^2+y^2+z^2$ and $w=yz+zx+xy$. **(15)**
- If $\vec{u}=4y\hat{i}+x\hat{j}-2z\hat{k}$, calculate the double integral $\iint(\nabla\times\vec{u})\cdot d\vec{s}$ over the hemisphere given by $x^2+y^2+z^2=a^2, z\ge 0$. **(15)**

- If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal. **(15)**
- Verify Gauss Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$ taken over the cube $0 \le x, y, z \le 1$. **(15)**

2010

- Find the directional derivative of $f(x,y) = x^2y^3 + xy$ at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x – axis. **(12)**
- Show that the vector field defined by the vector function $\vec{V} = xyz(yz\,\vec{i} + xz\,\vec{j} + xy\vec{k})$ is conservative. **(12)**
- Prove that $div\left(f\vec{V}\right) = f\left(div\vec{V}\right) + \left(grad\ f\right)\vec{V}$ where f is a scalar function. **(20)**
- Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}\,dA$ where $\vec{V} = x^2z\,\vec{i} + y\,\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane z = 4y. **(20)**
- Verify Green's theorem for ; $e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ the path of integration being the boundary of the square whose vertices are (0, 0), (π/2, 0), (π/2, π/2) and (0, π/2). **(20)**

2009

- Show that $div(grad\ r^n) = n(n+1)r^{n-2}$ Where $r = \sqrt{x^2 + y^2 + z^2}$. **(12)**
- Find the directional derivatives of –
  (i) $4xz^3 - 3x^2y^2z^2$ at (2, -1, 2) along z – axis;
  (ii) $x^2yz + 4xz^2$ at (1, -2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(12)**
- Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the field of force given by $\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$. **(20)**
- Using divergence theorem, evaluate $\iint_S \vec{A}.d\vec{S}$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(20)**
- Find the value of $\iint_S (\vec{\nabla} \times \vec{F}).\,d\vec{S}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane z = 0, when $\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$. **(20)**

2008

- Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2)
- Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).
- P.T $\nabla^2 f(r) = \frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = (x^2 + y^2 + z^2)^{1/2}$. Hence find f(r) such that $\nabla^2 f(r) = 0$.
- Show that for the space curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$ the curvature and torsion are same at every point.
- Evaluate $\int_c \vec{A}d\vec{r}$ along the curve $x^2 + y^2 = 1,\ z = 1$ from (0, 1, 1) to (1, 0,1) if $\vec{A} = (yz + 2x)\,\hat{i} + xz\,\hat{j} + (xy + 2z)\hat{k}$.
- Evaluate $\iint_s \vec{F}\cdot\hat{n}\,dS$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and 'S' is the surface of the cylinder bounded by $x^2 + y^2 = 4$, z = 0 and z = 3.

2007

- If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$

determine grad (r^{-1}) in terms of $\hat{r}$ and r.

- Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$
- For any constant vector $\vec{a}$ show that the vector represented by curl $(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z), measured from the origin.
- If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find the value(s) of n in order that $r^n\vec{r}$ may be (i) solenoidal or (ii) irrotational
- Determine $\int_C (y\,dx + z\,dy + x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2$, $x + y = 2a$ that starts from the point (2a, 0, 0) and goes at first below the z – plane.

2006

- Find the values of constant a, b, and c so that the directional of the function $f = axy^2 + byz + cz^2x^3$ at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to Z–axis.
- If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} - 7\hat{k},$ determine a vector $\vec{R}$ satisfying the vector equations $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}\cdot\vec{A} = 0$
- Prove that $r^n\vec{r}$ is an irrotational vector for any value of n , but is solenoidal only if n + 3 = 0 .
- If the unit tangent vector $\vec{t}$ and binormal $\vec{b}$ makes angles θ and ϕ respectively with a constant unit vector $\vec{a}$, prove that $\dfrac{\sin\theta}{\sin\phi}.\dfrac{d\theta}{d\phi} - \dfrac{k}{\tau}$
- Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane z = 0 and bounded by the lines x=0, y = 0 , x =a and y = a , a > 0.

2005

- Show that the volume of the tetrahedron ABCD is $\frac{1}{6}\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\cdot\overrightarrow{AD}.$ Hence find the volume of the tetrahedron with vertices (2, 2, 2) , (2, 0, 0) , (0, 2, 0) and (0, 0, 2).
- Prove that the curl of a vector field is independent of the choice of co – ordinates.
- The parametric equation of a circular helix is $\vec{r} = a\cos u\,\hat{i} + a\sin u\,\hat{j} + cu\,\hat{k}$; where 'c' is a constant and 'u' is a parameter.
- Find the unit tangent vector $\hat{t}$ at the point 'u' and the arc length measured from u = 0. Also find $\frac{d\hat{t}}{ds}$, where 'S' is the arc length.
- Show that $curl\left(\hat{k}\times grad\dfrac{1}{r}\right) + grad\left(\hat{k}\cdot grad\dfrac{1}{r}\right) = 0$ where r is the distance from the origin and $\hat{k}$ is the unit vector in the direction OZ.
- Find the curvature and the torsion of the space curve $x = a\left(3u - u^3\right), y = 3au^2,\ z = a\left(3u + u^3\right).$
- Evaluate $\oint\!\!\oint_S (x^3dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy)$ by Gauss divergence theorem, where S is the surface of the cylinde $x^2 + y^2 = a^2$ bounded by z =0 and z = b.

2004

- Show that if $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A}\times\vec{B}$ is solenoidal.
- Show that the Frenet – Serret formulae can be written in the form $\dfrac{d\vec{T}}{ds} = \vec{\omega}\times\vec{T}$, $\dfrac{d\vec{N}}{ds} = \vec{\omega}\times\vec{N}$ and $\dfrac{d\vec{B}}{ds} = \vec{\omega}\times\vec{B}$ Where, $\vec{\omega} = \tau\vec{T} + k\vec{B}$
- Prove the identity $\nabla\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{B}\cdot\nabla\right)\vec{A} + \left(\vec{A}\cdot\nabla\right)\vec{B} + \vec{B}\times\left(\nabla\times\vec{A}\right) + \vec{A}\times\left(\nabla\times\vec{B}\right)$
- Derive the identity $\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)d\forall = \iint_S (\phi\nabla\psi - \psi\nabla\phi).\hat{n}\,dS$, where V is the volume bounded by the closed surface S.
- Verify Stoke's theorem for

$\vec{f} = (2x-y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

2003

- ❖ Show that if $\vec{a}', \vec{b}'$ *and* $\vec{c}'$ are the reciprocals of the non – coplanar vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, then any vector $\vec{r}$ may be expressed as $\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')b + (\vec{r}.\vec{c}')c.$
- ❖ Prove that the divergence of a vector field is invariant w. r. t co – ordinate transformations.
- ❖ Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.
- ❖ Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A\times(\nabla\times A)$

  Where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.
- ❖ Find the radii of curvature and torsion at a point of intersection of the surfaces $x^2 - y^2 = c^2,\ y = x\tanh\left(\frac{z}{c}\right)$
- ❖ Evaluate $\iint_S curl\ \ A.dS$ where S is the open surface $x^2+y^2-4x+4z=0,\ z\ge 0$ and $A = (y^2+z^2-x^2)\hat{i} + (2z^2+x^2-y^2)\hat{j} + (x^2+y^2-3z^2)\hat{k}$

2002

- ❖ Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$. Show that $\vec{R}\times\frac{d\vec{R}}{dt} = \frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ where $r = |\vec{r}|$.
- ❖ Find the curvature K for the space curve $x = a\cos\theta,\ y = a\sin\theta,\ z = a\theta\tan\alpha$
- ❖ Show that curl (curl $\vec{v}$) = grad ($div\vec{v}$) - $\nabla^2\vec{v}$
- ❖ Let D be a closed and bounded region having boundary S. Further let 'f' be a scalar function having second order partial derivatives defined on it. Show that

  $$\iint_S (f\ grad\ f).\hat{n}\,dS = \iiint_V \left[|grad|^2 + f\nabla^2 f\right] d\forall$$

  Hence or otherwise evaluate $\iint_S (f\ grad\ f).\hat{n}\,dS$ for f = 2x + y +2z over $S \equiv x^2+y^2+z^2 = 4$
- ❖ Find the values of constants a, b, and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at (1, -1, 1) is in the direction parallel to y axis and has magnitude 6.

2001

- ❖ Find the length of the arc of the twisted curve $\vec{r} = (3t, 3t^2, 2t^3)$ from the point $t = 0$ to the point $t = 1$. Find also the unit tangent $\vec{t}$, unit normal $\vec{n}$ and the unit binormal $\vec{b}$ at $t = 1$.
- ❖ Show that curl $\frac{\vec{a}\times\vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a}.\vec{r})$ where $\vec{a}$ *is a* constant vector.
- ❖ Find the directional derivative of $f = x^2yz^3$ along $x = e^{-t},\ y = 1+2\sin t,\ z = t-\cos t$ at t = 0.
- ❖ Show that the vector field defined by $F = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find also the scalar 'u' such that F = grad u.
- ❖ Verify Gauss divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2+y^2=4$, z =0 & z = 3.

2000

- ❖ In what direction from the point (-1 , 1 , 1) is the directional derivative of $f = x^2yz^3$ a maximum? compute its magnitude.
- ❖ Show that

  (i) $(A+B).(B+C)\times(C+A) = 2A.B\times C$

  (ii) $\nabla\times(A\times B) = (B.\nabla)A - B(\nabla.A) - (A\nabla)B + A(\nabla.B)$

  (1990)
- ❖ Evaluate $\iint_S F.\hat{n}\,dS$ where $F = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$

and S is the surface of the parallelopiped bounded by $x=0$, $y=0$, $z=0$, $x=2$, $y=1$ and $z=3$.

1999

- If $\vec{a},\vec{b},\vec{c}$ are the position vectors A, B, C prove that $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane ABC.
- If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$, find $\nabla\times\vec{F}$.
- Evaluate $\int_c\left(e^{-x}\sin y\,dx+e^{-x}\cos y\,dy\right)$; (by Green's theorem), where 'C' is the rectangle whose vertices are $(0,0)$, $(\pi,0)$ $(\pi,\pi/2)\&(0,\pi/2)$.
- If X, Y, Z are the components of a contra variant vector in rectangular cartesian co-ordinates x,y,z in a three dimensional space, show that the components of the vector in cylindrical co-ordinates

  r,θ,Z are $X\cos\theta+Y\sin\theta,\frac{-x}{\mathrm{r}}\sin\theta+\frac{y}{\mathrm{r}}\cos\theta,Z$

1998

- If r_1 and r_2 are the vectors joining the fixed points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ respectively to a variable point P (x , y , z), then find the values of grad $(r_1\cdot r_2)$ and curl $(r_1\times r_2)$
- Show that $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ if either $\vec{b}=0$ (or any other vector is '0') or $\vec{c}$ is collinear with $\vec{a}$ or $\vec{b}$ is orthogonal to $\vec{a}$ and $\vec{c}$ (both).

1997

- Prove that if $\vec{A},\vec{B}$ and $\vec{C}$ are three given non coplanar vectors, then any vector $\vec{F}$ can be put in the form $\vec{F}=\alpha\vec{B}\times\vec{C}+\beta\vec{C}\times\vec{A}+\gamma\vec{A}\times\vec{B}$. For a given $\vec{F}$ determine α ,β ,γ.
- Verify Gauss theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4$, and $z=0$ and $z=3$.

1996

- If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$, show that

  (i) $\vec{r}\times grad\ f(r)=0$

  (ii) $div\ (r^n\vec{r})=(n+3)r^n$
- Verify Gauss divergence theorem for $\vec{F}=xy\hat{i}+z^2\hat{j}+2yz\hat{k}$, on the tetrahedron $x=y=z=0$, $x+y+z=1$

1994

- If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$. evaluate $\iint_S(\nabla\times\vec{F}).\hat{n}\,dS$.

1993

- Evaluate $\iint_s\nabla\times\vec{F}.\hat{n}\,ds$, where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k}$.

1992

- If $\vec{f}(x,y,z)=(y^2+z^2)\,\hat{i}+(z^2+x^2)\,\hat{j}+(x^2+y^2)\hat{k}$ then calculate $\int_C\vec{f}.d\vec{x}$ where 'C' consists of
  - (i) The line segment from (0,0,0) to (1,1,1)
  - (ii) The three line segments AB, BC and CD, where A, B, C and D are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)
  - (iii) The curve $\vec{x}=u\hat{i}+u^2\hat{j}+u^3\hat{k}$, u from 0 to 1.
- If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

  (i) $div\left\{\vec{x}\times\left(\vec{a}\times\vec{x}\right)\right\}=-2\vec{x}.\vec{a}$

  (ii) $div\left\{\left(\vec{a}\times\vec{x}\right)\times\left(\vec{b}\times\vec{x}\right)\right\}=2\vec{a}.\left(\vec{b}\times\vec{x}\right)-2\vec{b}.\left(\vec{a}\times\vec{x}\right)$

1991

- If ϕ be a scalar point function and F be a vector point function, show that the components of F normal and tangential to surface $\phi=0$ at any point there of are $\frac{(F.\nabla\phi)\nabla\phi}{(\nabla\phi)^2}$ and $\frac{\nabla\phi\times(F\times\nabla\phi)}{(\nabla\phi)^2}$

❖ Find the value of $\int$ curl F. dS taken over the portion of the surface $x^2+y^2-2ax+az=0$, for which $z\geq 0$, when $F=(y^2+z^2-x^2)\,\hat{i}+(z^2+x^2-y^2)\,\hat{j}+(x^2+y^2-z^2)\hat{k}$.

1989

❖ Define the curl of a vector point function

❖ Prove that $\nabla\times(\frac{\vec{r}}{r^2})=0$ where $\vec{r}=(x,y,z)$ and $r=|\vec{r}\,|$.

1988

❖ Define the divergence of a vector point function, prove that $div\,(\vec{u}\times\vec{v})=\vec{v}.curl\,\vec{u}-\vec{u}.curl\,\vec{v}$. ***(1986)***

❖ Using Gauss divergence theorem, evaluate $\iint_S (x\hat{i}+y\hat{i}+z^2\hat{k}).\hat{n}\,ds$ where S is the closed surface bounded by the cone $x^2+y^2=2$ and the plane Z=1 and $\hat{n}$ is the outward unit normal to S.

1987

❖ Show that for a vector field $\vec{f}$, curl (curl $\vec{f}$) = grad (div $\vec{f}$) - $\nabla^2\vec{f}$.

❖ If $\vec{r}$ is the position vector to a point whose distance from the origin is r, prove that $div\,\vec{f}=0$ if $\vec{f}=\frac{\vec{r}}{r^3}$.

❖ Prove that for a three vectors $\vec{a},\vec{b},\vec{c}$ $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\vec{b}\left(\vec{a}.\vec{c}\right)-\vec{c}\left(\vec{a}.\vec{b}\right)$ and explain its geometric meaning. ***(1990)***

1986

❖ Let $\vec{a},\vec{b}$ be given vectors in the three dimensional Euclidean space E_3 and let $\phi(\vec{x})$ be a scalar field of the vectors $\vec{x}$ also of E_3.
If $\phi(\vec{x})=(\vec{x}\times\vec{a}).(\vec{x}\times\vec{b})$,
show that grad
$\phi(i.e,\nabla\phi(\vec{x}))=\vec{b}\times(\vec{x}\times\vec{a})+\vec{a}\times(\vec{x}\times\vec{b})$.

❖ If $\vec{f},\vec{g}$ are two vector fields in E_3 and if 'div', 'curl' are defined on an open set $S\subset E_3$ show that $div(\vec{f}\times\vec{g})=\vec{g}.\,curl\,\vec{f}-\vec{f}.curl\,\vec{g}$. ***(1988)***

1985

❖ If P,Q,R are points (3,-2,-1) , (1,3,4) , (2,1,-2) respectively. Find the distance from P to the plane OQR, where 'O' is the origin.

❖ Find the angle between the tangents to the curve $\vec{r}=t^2\hat{i}-2t\,\hat{j}+t^3\hat{k}$ at the points t=1 and t =2

❖ Find div F and curl F, where $F=\nabla(x^3+y^3+z^3-3xyz)$

1983

❖ Prove that curl (curl F) = grad div F - $\nabla^2 F$.