IFoS
PREVIOUS YEARS QUESTIONS (2017-2000) SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2017

- Solve $(2D^3 - 7D^2 + 7D - 2)\,y = e^{-8x}$ where $D = \frac{d}{dx}$. **(8)**
- Solve the differential equation
$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4.$$ **(8)**
- Solve the differential equation
$$\left(\frac{dy}{dx}\right)^2 + 2\cdot\frac{dy}{dx}\cdot y\cot x = y^2.$$ **(15)**
- Solve the differential equation
$$e^{3x}\left(\frac{dy}{dx} - 1\right) + \left(\frac{dy}{dx}\right)^3 e^{2y} = 0.$$ **(10)**
- Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by using the method of variation of parameter. **(10)**

2016

- Obtain the curve which passes through (1, 2) and has a slope $= \frac{-2xy}{x^2+1}$. Obtain one asymptote to the curve. **(8)**
- Solve the dE to get the particular integral of
$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2\cos x.$$ **(8)**
- Using the method of variation of parameters, solve
$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x.$$ **(10)**
- Obtain the singular solution of the differential equation
$$y^2 - 2pxy + p^2\left(x^2 - 1\right) = m^2,\; p = \frac{dy}{dx}.$$ **(10)**
- Solve the differential equation **(10)**
$$\frac{dy}{dx} - y = y^2\left(\sin x + \cos x\right).$$

2015

- Reduce the differential equation $x^2p^2 + yp(2x+y) + y^2 = 0,\; p = \frac{dy}{dx}$ to Clairaut's form. Hence, find the singular solution of the equation. **(8)**
- Solve the differential equation
$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$ **(8)**
- Solve $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin x^2$ by changing the independent variable. **(10)**
- Solve $(D^4 + D^2 + 1)y = e^{-x/2}\cos\left(\frac{x\sqrt{3}}{2}\right)$, where $D = \frac{d}{dx}$. **(10)**

2014

- Solve the differential equation
$$y = 2px + p^2y,\; p = \frac{dy}{dx}$$
and obtain the non-singular solution **(8)**
- Solve
$$\frac{d^4y}{dx^4} - 16y = x^4 + \sin x.$$ **(8)**

❖ Solve the following differential equation

$$\frac{dy}{dx}=\frac{2y}{x}+\frac{x^3}{y}+x\tan\frac{y}{x^2}.$$ **(10)**

❖ Solve by the method of variation of parameters

$y''+3y'+2y=x+\cos x.$ **(10)**

❖ Solve the D.E.

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x.$$ **(10)**

2013

❖ Solve

$$\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$$ **(8)**

❖ Solve the differential equation

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$$

by changing the dependent variable. **(13)**

❖ Solve

$$(D^3+1)y=e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x\right)$$

where $D=\frac{d}{dx}$. **(13)**

❖ Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2}-y=2(1+e^x)^{-1}$$ **(13)**

2012

❖ Solve $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y.$ ***(8)***

❖ Solve and find the singular solution of

$x^3p^2+x^2py+a^3=0$ ***(8)***

❖ Solve: $x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2=0$ ***(10)***

❖ Solve $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2\cos x.$ ***(10)***

❖ Solve $x=y\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2$ ***(10)***

❖ Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=(1-x)^{-2}$ ***(10)***

2011

❖ Find the family of curves whose tangents form an angle $\pi/4$ With hyperbolas $xy=c.$ **(10)**

❖ Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=\sec x.e^x.$ **(10)**

❖ Solve $p^2+2py\cot x=y^2$ Where $p=\frac{dy}{dx}$. **(10)**

❖ Solve $\left\{x^4D^4+6x^3D^3+9x^2D^2+3xD+1\right\}y=\left(1+\log x\right)^2,$

Where $D=\frac{d}{dx}.$ **(15)**

❖ Solve $(D^4+D^2+1)y=ax^2+be^{-x}\sin 2x,$ where $D\equiv\frac{d}{dx}$ **(15)**

2010

❖ Show that $\cos(x+y)$ is an integrating factor of

$y\,dx+\left[y+\tan(x+y)\right]dy=0.$

Hence solve it **(8)**

❖ Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ **(8)**

❖ Solve the following differential equation

$$\frac{dy}{dx}=\sin^2(x-y+6)$$ **(8)**

❖ Find the general solution of

$$\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\left(x^2+1\right)y=0$$ **(12)**

❖ Solve

$$\left(\frac{d}{dx}-1\right)^2\left(\frac{d^2}{dx^2}+1\right)^2y=x+e^x$$ **(10)**

❖ Solve by the method of variation of parameters the following eqaution

$$\left(x^2-1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\left(x^2-1\right)^2 \quad \textbf{(10)}$$

2009

- Solve $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$ **(10)**
- Find the $2nd$ order ODE for which e^x and x^2e^x are solutions. **(10)**
- Solve $\left(y^3-2yx^2\right)dx+\left(2xy^2-x^3\right)dy=0$. **(10)**
- Solve $\left(\frac{dy}{dx}\right)^2-2\frac{dy}{dx}\cosh x+1=0.$ **(8)**
- Solve $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=x^2e^{-x}$ **(10)**
- Show that e^{x^2} is a solution of

  $$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-2\right)y=0. \quad \textbf{(12)}$$

  Find a second independent solution.

2008

- Show that the functions $y_1(x)=x^2$ *and* $y_2(x)=x^2\log_e x$ are linearly independent obtain the differential equation that has $y_1(x)$ *and* $y_2(x)$ as the independent solutions. **(10)**
- Solve the following ordinary differential equation of the second degree :

  $$y\left(\frac{dy}{dx}\right)^2+\left(2x-3\right)\frac{dy}{dx}-y=0 \quad \textbf{(10)}$$

- Reduce the equation $\left(x\frac{dy}{dx}-y\right)\left(x-y\frac{dy}{dx}\right)=2\frac{dy}{dx}$ to clairaut's form and obtain there by the singular integral of the above equation. **(10)**
- Solve

  $$\left(1+x\right)^2\frac{d^2y}{dx^2}+\left(1+x\right)\frac{dy}{dx}+y=4\cos\log_e\left(1+x\right) \quad \textbf{(10)}$$

- Find the general solution of the equation

  $$\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-\left(1-\cot x\right)y=e^x\sin x. \quad \textbf{(10)}$$

2007

- Find the orthogonal trajectories of the family of the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, λ being a parameter. **(10)**
- Show that e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. Find the general solution when $y(0)=0$ *and* $\left.\frac{dy}{dx}\right]_0=1$ **(10)**
- Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbola $xy=c.$ **(10)**
- Apply the method of variation of parametes to solve $\left(D^2+a^2\right)y=\operatorname{cosec} ax$ **(10)**
- Find the general solution of $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+3y=0$ solution of it. **(10)**

2006

- From $x^2+y^2+2ax+2by+c=0$, derive differential equation not containing, a,b *or* c. **(10)**
- Discuss the solution of the differential equation

  $$y^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]=a^2 \quad \textbf{(10)}$$

- Solve $x\frac{d^2y}{dx^2}+\left(1-x\right)\frac{dy}{dx}-y=e^x$ **(10)**
- Solve $\frac{d^4y}{dx^4}-y=x\sin x$ **(10)**
- Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$ **(10/2008)**
- Reduce

  $$xy\left(\frac{dy}{dx}\right)^2-\left(x^2+y^2+1\right)\frac{dy}{dx}+xy=0$$

to clairaut's form and find its singular solution. **(10)**

2005

- Form the differential equation that represents all parabolas each of which has latus rectum $4a$ and Whose are parallel to the x-axis. **(10)**
- (i) The auxiliary polynomial of a certain homogenous linear differential equation with constant coefficients in factored form is

  $$P(m)=m^4(m-2)^6\ \left(m^2-6m+25\right)^3.$$

  What is the order of the differential equation and write a general solution ?

  (ii) Find the equation of the one-parameter family of parabolas given by $y^2=2cx+c^2$, C real and show that this family is self-orthogonal. **(10)**
- Solve and examine for singular solution the following equation $P^2\left(x^2-a^2\right)-2pxy+y^2-b^2=0$ **(10)**
- Solve the differential equation $\dfrac{d^2y}{dx^2}+9y=\sec 3x$ **(10)**
- Given $y=x+\dfrac{1}{x}$ is one solution solve the differential equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-y=0$ reduction of order method. **(10)**
- Find the general solution of the defferential equation $\dfrac{d^2y}{dx^2}-2y\dfrac{dy}{dx}-3y=2e^x-10\sin x$ by the method of undertermined coefficients. **(10)**

2004

- Dertermine the family of orthgonal trajectories of the family $y=x+ce^{-x}$ **(10)**
- Show that the solution curve satisfying $\left(x^2-xy\right)\ y'=y^3$ Where $y\to 1\ as\ x\to\infty$, is a conic section. Indentify the curve. **(10)**
- Solve $(1+x)^2y''+(1+x)y'+y=4\cos(\ln(1+x))$, $y(0)=1, y(e-1)=\cos 1.$ **(10)**
- Obtain the general solution of $y''+2y'+2y=4e^{-x}x^2\ \sin x.$ **(10)**
- Find the general solution of $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ **(10)**
- Obtain the general solution of $\left(D^4+2D^3-D^2-2D\right)$ $y=x+e^{2x}$, Where $D_y=\dfrac{dy}{dx}$. **(10)**

2003

- Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c=0$ Where g is a parameter. **(10)**
- Find the three solutions of $\dfrac{d^3y}{dx^3}-2\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}+2y=0$ Which are linealy independent on every real interval. **(10)**
- Solve and examine for singular solution: $y^2-2pxy+p^2\left(x^2-1\right)=m^2.$ **(10)**
- Solve $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$ **(10)**
- Given $y=x$ is one solutions of $\left(x^3+1\right)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$ find another linearly independent solution by reducing order and write the general solution. **(10)**
- Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}+a^2y=\sec ax,$ a real. **(10)**

2002

- If $(D-a)^4e^{nx}$ is denoted by z, prove that $z\dfrac{\partial z}{\partial n},\dfrac{\partial^2 z}{\partial n^2},\dfrac{\partial^3 z}{\partial n^3}$ all vanish when $n=a$. Hence show that $e^{nx}, xe^{nx}, x^2e^{nx}$, x^3nx are all solutions of $(D-a)^4 y = 0$. Here D Stands for $\frac{d}{dx}$. **(10)**

- Solve $4xp^2 - (3x+1)^2 = 0$ and examine for singular solutions and extraneous loci. Interpret the results geometrically. **(10)**
- (i) Form the differential equation whose primitive is

$$y = A\left(\sin x + \frac{\cos x}{x}\right) + B\left(\cos x - \frac{\sin x}{x}\right)$$

(ii) Prove that the orthogonal trajectory of system of parabolas belongs to the system itself. **(10)**

- Using variation of parameters solve the differential equation

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + \left(4x^2-1\right)y = -3e^{x^2}\sin 2x. \quad \textbf{(10)}$$

- (i) Solve the equation by finding an integrating factor of $(x+2)\sin y dx + x\cos y dy = 0.$

(ii) Verify that $\phi(x) = x^2$ is a solution of $y'' - \frac{2}{x^2}y = 0$ and find a second independent solution. **(10)**

- Show that the solution of $\left(D^{2n+1}-1\right)y = 0,$ consists of Ae^x and n paris of terms of the form $e^{ax}\left(b_r\cos\alpha x + c_r\sin\alpha x\right)$, Where $a = \cos\frac{2\pi r}{2n+1}$ and $\alpha = \sin\frac{2\pi r}{2n+1}$, $r = 1,2.....,n$ and b_r, c_r are arbitrary constants. **(10)**

2001

- A constant coefficient differential equation has auxiliary eqution expressible in factored form as $P(m) = m^3(m-1)^2\left(m^2+2m+5\right)^2$. What is the order of the differential equation and find its general solution. **(10)**
- Solve $x^2\left(\frac{dy}{dx}\right)^2 + y(2x+y)\frac{dy}{dx} + y^2 = 0$ **(10)**
- Using differential equations show that the system of confocal conics given by $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1,$ is self othogonal. **(10)**
- Solve $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ given that $y = e^{a\sin x^{-1}}$ is one solution of this equation. **(10)**
- Find a general solution $y^n + y = \tan x, -\pi/2 < x < \pi/2$ by variation of parameters. **(10)**

2000

- Solve $\left(x^2+y^2\right)(1+P)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$ $P \equiv \frac{dy}{dx}$. Interpret geometrically the factors in the P- and C-discriminants of the equation $8p^3x = y\left(12p^2-9\right)$ **(20)**
- Solve

(i) $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0$

(ii) $\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x.$

by varying parameters. **(20/2007)**