IAS
PREVIOUS YEARS QUESTIONS (2017-2001) SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS

2017

- Find the differential equation representing all the circles in the x-y plane. **(10)**
- Suppose that the streamlines of the fluid flow are given by a family of curves xy=c. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. **(10)**
- Solve the following simultaneous linear differential equations: $(D+1)y = z+e^x$ and $(D+1)z=y+e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. **(08)**
- If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? **(08)**
- Consider the differential equation $xyp^2-(x^2+y^2-1)p+xy=0$ where $p=\frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p'=\frac{dv}{du}$. Hence, or otherwise solve the equation. **(10)**
- Solve the following initial value differential equations: $20y''+4y'+y=0$, $y(0)=3.2$ and $y'(0)=0$. **(07)**
- Solve the differential equation:
$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2).$$ **(09)**
- Solve the following differential equation using method of variation of parameters:
$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$ **(08)**
- Solve the following initial value problem using Laplace transform:
$$\frac{d^2y}{dx^2}+9y=r(x), y(0)=0, y'(0)=4$$
where $r(x)=\begin{cases}8\sin x & if \quad 0<x<\pi \\ 0 & if \quad x\geq\pi\end{cases}$ **(17)**

2016

- Find a particular integral of
$$\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}.$$ **(10)**
- Solve :
$$\frac{dy}{dx}=\frac{1}{1+x^2}\left(e^{\tan^{-1}x}-y\right)$$ **(10)**
- Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **(10)**
- Solve :
$\{y(1-x\tan x)+x^2\cos x\}\,dx-xdy=0$ **(10)**
- Using the method of variation of parameters, solve the differentail equation
$$(D^2+2D+1)y=e^{-x}\log(x), \left[D=\frac{d}{dx}\right]$$ **(15)**
- find the genral solution of the eqution
$$x^2\frac{d^3y}{dx^3}-4x\frac{d^2y}{dx^2}+6\frac{dy}{dx}=4.$$ **(15)**
- Using lapalce transformation, solve the following :
$y''-2y'-8y=0, y(0)=3, y'(0)=6$ **(10)**

2015

- Solve the differential equation :
$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1.$$ **(10)**

- Sove the differential equation :
  $(2xy^4e^y + 2xy^3 + y)\, dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ **(10)**

- Find the constant a so that $(x + y)^a$ is the Integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential eauation. **(12)**

- (i) Obtain Laplace Inverse transform of
  $$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$
  (ii) Using Laplace transform, solve
  y'' + y = t, y(0) = 1, y'(0 = –2. **(12)**

- Solve the differential equation
  $x = py - p^2$ where $p = \dfrac{dy}{dx}$

- Solve :
  $$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x).$$

2014

- Justify that a differential equation of the form :
  $[y + x\,f(x^2 + y^2)]\, dx+[y\,f(x^2 + y^2) - x]\, dy = 0$,
  where $f(x^2 + y^2)$ is an arbitrary function of (x^2+y^2), is not an exact differential equation and $\dfrac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2) = (x^2+y^2)^2$. **(10)**

- Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. **(10)**

- Solve by the method of variation of parameters :
  $$\frac{dy}{dx} - 5y = \sin x$$ **(10)**

- Solve the differential equation : **(20)**
  $$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log_e x)$$

- Solve the following differential equation : **(15)**
  $$x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x},$$
  when e^x is a solution to its corresponding homogeneous differential equation.

- Find the sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $(x^2 + xy)\, dx + (y^2+xy)\, dy = 0$ and solve it. **(15)**

- Solve the initial value problem
  $$\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t,\; y(0)=0, y'(0)=0$$
  by using Laplace-transform

2013

- y is a function of x, such that the differential coefficient $\dfrac{dy}{dx}$ is equal to $\cos(x + y) + \sin(x + y)$.
  Find out a relation between x and y, which is free from any derivative/differential. **(10)**

- Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a\sin n\theta$, (r, θ) being the plane polar coordinates. **(10)**

- Solve the differential equation
  $(5x^3 + 12x^2 + 6y^2)\, dx + 6xydy = 0$. **(10)**

- Using the method of variation of parameters, solve the differential equation $\dfrac{d^2y}{dx^2} + a^2y = \sec ax$. **(10)**

- Find the general solution of the equation **(15)**
  $$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = \ln x\,\sin(\ln x).$$

- By using Laplace transform method, solve the differential equation $(D^2 + n^2)\, x = a\sin(nt + \alpha)$, $D^2 = \dfrac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\dfrac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **(15)**

2012

- Solve $\dfrac{dy}{dx}=\dfrac{2xy\,e^{(x/y)^2}}{y^2(1+e^{(x/y)^2})+2x^2e^{(x/y)^2}}$ **(12)**

- Find the orthogonal trajectories of the family of curves $x^2 + y^2 = ax$. **(12)**

- Using Laplace transforms, solve the intial value problem $y''+2y'+y=e^{-t}, y(0)=-1, y'(0)=1$ **(12)**
- Show that the differential equation $$(2xy\log y)dx+\left(x^2+y^2\sqrt{y^2+1}\right)dy=0$$ is not exact. Find an integrating factor and hence, the solution of the equation. **(20)**
- Find the general solution of the equation $$y'''-y''=12x^2+6x.$$ **(20)**
- Solve the ordinary differential equation $$x(x-1)y''-(2x-1)y'+2y=x^2(2x-3)$$ **(20)**

2011

- Obtain the soluton of the ordinary differential equation $\frac{dy}{dx}=(4x+y+1)^2$, if $y(0)=1$. **(10)**
- Determine the orthogonal trajectory of a family of curves represented by the polar equation $r=a(1-\cos\theta)$, (r,θ) being the plane polar coordinates of any point. **(10)**
- Obtain Clairaut's orm of the differential equation $\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+y\right)=a^2\frac{dy}{dx}$. Also find its general solution. **(15)**
- Obtain the general solution of the second order ordinary differential equation $y''-2y'+2y=x+e^x\cos x$, where dashes denote derivatives w.r. to x. **(15)**
- Using the method of variation of parameters, solve the second order differedifferential equation $$\frac{d^2y}{dx^2}+4y=\tan 2x.$$ **(15)**
- Use Laplace transform method to solve the following initial value problem: $$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t,\ x(0)=2 \text{ and } \left.\frac{dx}{dt}\right|_{t=0}=-1$$ **(15)**

2010

- Consider the differential equation $$y'=\alpha x, x>0$$ where α is a constant. Show that–
  (i) if $\phi(x)$ is any solution and $\Psi(x)=\phi(x)e^{-\alpha x}$, then $\Psi(x)$ is a constant;
  (ii) if $\alpha<0$, then every solution tends to zero as $x\to\infty$. **(12)**
- Show that the diffrential equation $$(3y^2-x)+2y(y^2-3x)y'=0$$ admits an integrating factor which is a function of $(x+y^2)$. Hence solve the equation. **(12)**
- Verify that $$\frac{1}{2}(Mx+Ny)d(\log_e(xy))+\frac{1}{2}(Mx-Ny)d(\log_e(\tfrac{x}{y}))$$ $= M\,dx + N\,dy$
  Hence show that–
  (i) if the differential equation $M\,dx+N\,dy=0$ is homogeneous, then $(Mx+Ny)$ is an integrating factor unless $Mx+Ny\equiv 0$;
  (ii) if the differential equation $Mdx+Ndy=0$ is not exact but is of the form $f_1(xy)y\,dx+f_2(xy)x\,dy=0$ then $(Mx-Ny)^{-1}$ is an integrating factor unless $Mx-Ny\equiv 0$. **(20)**
- Show that the set of solutions of the homogeneous linear differential equation $$y'+p(x)y=0$$ on an interval $I=[a,b]$ forms a vector subspace W of the real vector space of continous functions on I. what is the dimension of W?. **(20)**
- Use the method of undetermined coefficiens to find the particular solution of $y''+y=\sin x+(1+x^2)e^x$ and hence find its general solution. **(20)**

2009

- Find the Wronskian of the set of functions $$\{3x^3, |3x^3|\}$$

on the interval $[-1, 1]$ and determine whether the set is linearly dependent on $[-1, 1]$. **(12)**

- Find the differential equation of the family of circles in the xy-plane passing through $(-1, 1)$ and $(1, 1)$. **(20)**

- Fidn the inverse Laplace transform of

$$F(s) = \ln\left(\frac{s+1}{s+5}\right).$$ **(20)**

- Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1.$ **(20)**

2008

- Solve the differential equation

$$ydx + \left(x + x^3y^2\right)dy = 0.$$ **(12)**

- Use the method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = -x^4 \sin x$. **(12)**

- Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1$, $y'(0) = -1$. **(15)**

- Solve the differential equation

$$x^3y'' - 3x^2y' + xy = \sin(\ln x) + 1.$$ **(15)**

- Solve the equation $y - 2xp + yp^2 = 0$ where $p = \frac{dy}{dx}$. **(15)**

2007

- Solve the ordinary differential equation

$$\cos 3x \frac{dy}{dx} - 3y \sin 3x = \frac{1}{2}\sin 6x + \sin^2 3x, 0 < x < \frac{\pi}{2}.$$ **(12)**

- Find the solution of the equation

$$\frac{dy}{y} + xy^2dx = -4xdx.$$ **(12)**

- Determine the general and singular solutions of the equation $y = x\frac{dy}{dx} + a\frac{dy}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-1/2}$ 'a' being a constant. **(15)**

- Obtain the general solution of $\left[D^3 - 6D^2 + 12D - 8\right]$

$$y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right), \text{ where } D = \frac{d}{dx}.$$ **(15)**

- Solve the equation $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$. **(15)**

- Use the method of variation of parameters to find the general solution of the equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x.$$ **(15)**

2006

- Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbolas $xy=c, c > 0$. **(12)**

- Solve the differential eqaution

$$\left(xy^2 + e^{-1/x^3}\right)dx - x^2y\, dy = 0.$$ **(12)**

- Solve $\left(1 + y^2\right) + \left(x - e^{-\tan^{-1}y}\right)\frac{dy}{dx} = 0$. **(15)**

- Solve the equation $x^2p^2 + yp(2x + y) + y^2 = 0$ using the substituion $y = u$ and $xy = v$ and find its singular solution, where $p = \frac{dy}{dx}$. **(15)**

- Solve the differential equation

$$x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right).$$ **(15)**

- Solve the differential equation

$$\left(D^2 - 2D + 2\right)y = e^x \tan x, \text{ where } D = \frac{d}{dx},$$

by the method of variation of parameters. **(15)**

2005

- Find the orthogonal trajectory of a system of co-axial circles $x^2+y^2+2gx+c=0$, where g is the parameter. **(12)**

- Solve $xy\frac{dy}{dx} = \sqrt{x^2 - y^2 - x^2y^2 - 1}$. **(12)**

- Solve the differential equation $(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)y = \frac{1}{x+1}$. **(15)**

- Solve the differential equation $(x^2+y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp)+(x+yp)^2=0$, where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **(15)**
- Solve the differential equation $(\sin x - x\cos x)\, y'' - x\sin x\, y' + y\sin x = 0$ given that $y=\sin x$ is a solution of this equation. **(15)**
- Solve the differential equation

  $x^2y''-2xy'+2y = x\log x, x>0$

  by variation of parameters. **(15)**

2004

- Find the solution of the following differential equation $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$. **(12)**
- Solve $y(xy+2x^2y^2)\,dx + x(xy-x^2y^2)\,dy=0$. **(12)**
- Solve $\left(D^4-4D^2-5\right)y=e^x(x+\cos x)$. **(15)**
- Reduce the equation $(px-y)(py+x)=2p$ where $p=\frac{dy}{dx}$ to Clairaut's equation and hence solve it. **(15)**
- Solve $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$. **(15)**
- Solve the following differential equation

  $(1-x^2)\frac{d^2y}{dx^2}-4x\frac{dy}{dx}-(1+x^2)y=x$. **(15)**

2003

- Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal. **(12)**
- Solve $x\frac{dy}{dx}+y\log y = xye^x$. **(12)**
- Solve $(D^5-D)\,y=4(e^x+\cos x+x^3)$, where $D=\frac{d}{dx}$. **(15)**
- Solve the differential equation $(px^2+y^2)(px+y)=(p+1)^2$ where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions. **(15)**
- Solve $(1+x)^2y''+(1+x)y'+y=\sin 2\left[\log(1+x)\right]$. **(15)**
- Solve the differential equation

  $x^2y''-4xy'+6y=x^4\sec^2 x$

  by variation of parameters. **(15)**

2002

- Solve $x\frac{dy}{dx}+3y=x^3y^2$. **(12)**
- Find the values of λ for which all solutions of $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x\to\infty$. **(12)**
- Find the value of constant λ such that the following differential equation becomes exact.

  $\left(2xe^y+3y^2\right)\frac{dy}{dx}+\left(3x^2+\lambda e^y\right)=0$

  Further, for this value of λ, solve the equation. **(15)**
- Solve $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$. **(15)**
- Using the method of variation of parameters, find the solution of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ with $y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$. **(15)**
- Solve $(D-1)(D^2-2D+2)\,y=e^x$ where $D=\frac{d}{dx}$. **(15)**

2001

- A continuous function y(t) satisfies the differential equation

  $$\frac{dy}{dt}=\begin{cases}1+e^{1-t}, 0\le t<1\\ 2+2t-3t^2, 1\le t\le 5\end{cases}$$

  If $y(0)=-e$, find $y(2)$. **(12)**
- Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log_e x$. **(12)**

- Solve $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$. **(15)**
- Find the general solution of $ayp^2+(2x-b)p-y=0$, $a>o$. **(15)**
- Solve $(D^2+1)^2 y = 24x\cos x$ given that $y=Dy=D^2y=0$ and $D^3y = 12$ when $x = 0$. **(15)**
- Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$. **(15)**

2000

- Show that $3\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-8y=0$ has an integral which is a polynomial in x. Deduce the general solution. **(12)**
- Reduce $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, where P, Q, R are functions of x, to the normal form.

  Hence solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$. **(15)**
- Solve the differential equation $y = x-2ap+ap^2$. Fnd the singular solution and interpret it geometrically. **(15)**
- Show that $(4x+3y+1)dx+(3x+2y+1)dy = 0$ represents a family of hyperbolas with a common axis and tangent at the vertex. **(15)**
- Solve $x\frac{dy}{dx}-y=(x-1)\left(\frac{d^2y}{dx^2}-x+1\right)$ by the method of variation of parameters. **(15)**

1999

- Solve the differential equation

  $$\frac{xdx+ydy}{xdy-ydx}=\left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$$
- Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$.
- By the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec(ax)$.

1998

- Solve the differential equation $xy-\frac{dy}{dx}=y^3e^{-x^2}$
- Show that the equation $(4x+3y+1)\,dx + (3x+2y+1)\,dy = 0$ represents a family of hyperbolas having as asymptotes the lines $x+y = 0$; $2x+y+1=0$. **(1992)**
- Solve the differential equation $y = 3px + 4p^2$.
- Solve $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{4x}(x^2+9)$.
- Solve the differential equation

  $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\sin x$.

1997

- Solve the initial value problem $\frac{dy}{dx}=\frac{x}{x^2y+y^3}$, $y(0)=0$.
- Solve $(x^2-y^2+3x-y)\,dx + (x^2-y^2+x-3y)dy = 0$.
- Solve $\frac{d^4y}{dx^4}+6\frac{d^3y}{dx^3}+11\frac{d^2y}{dx}+6\frac{dy}{dx}=20e^{-2x}\sin x$
- Make use of the transformation $y(x) = u(x)\sec x$ to obtain the solution of $y''-2y'\tan x+5y=0$; $y(0)=0$; $y'(0)=\sqrt{6}$.
- Solve $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$; $y(0)=0$ and $y'(0)=2$.

1996

- Solve $x^2(y-px) = yp^2$; $\left(p=\frac{dy}{dx}\right)$.
- Solve $y\sin 2x\,dx - (1+y^2+\cos^2 x)\,dy = 0$.
- Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$. Find the value of y when $x=\pi/2$, if it is given that y=3 and

$\frac{dy}{dx}=0$ when x=0.

- Solve $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}=x^2+3e^{2x}+4\sin x$.
- Solve $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$.

1995

- Solve $(2x^2+3y^2-7)x\,dx-(3x^2+2y^2-8)y\,dy=0$.
- Test whether the equation $(x+y)^2\,dx-(y^2-2xy-x^2)\,dy=0$ is exact and hence solve it.
- Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. **(1998)**
- Determine all real valued solutions of the equation

  $y'''-iy''+y'-iy=0$, $y'=\frac{dy}{dx}$.
- Find the solution of the equation $y''+4y=8\cos 2x$ given that $y=0$ and $y'=2$ when x = 0.

1994

- Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.
- Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)$ is a function of x only say, f(x), then $F(x)=e^{\int f(x)dx}$ is an integrating factor of Pdx + Qdy = 0.
- Find the family of curves whose tangent form angle $\pi/4$ with the hyperbola xy = c.
- Transform the differential equation

  $\frac{d^2y}{dx^2}\cos x+\frac{dy}{dx}\sin x-2y\cos^3 x=2\cos^5 x$ into one having z an independent variable where z = sin x and solve it.
- If $\frac{d^2x}{dt^2}+\frac{g}{b}(x-a)=0$, (a, b and g being positive constants) and $x=a'$ and $\frac{dx}{dt}=0$ when t=0, show that

  $x=a+(a'-a)\cos\sqrt{\frac{g}{b}}t$.
- Solve $(D^2-4D+4)y=8x^2e^{2x}\sin 2x$, where, $D=\frac{d}{dx}$.

1993

- Show that the system of confocal conics

  $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
- Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x-x\sin y\}dy=0$.
- Solve $\frac{d^2y}{dx^2}+w_0^2y=a\cos wt$ and discuss the nature of solution as $w\to w_0$.
- Solve $(D^4+D^2+1)y=e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right)$.

1992

- By eliminating the constants a, b obtain the differential equation of which $xy=ae^x+be^{-x}+x^2$ is a solution.
- Find the orthogonal trajectories of the family of semicubical parabolas $ay^2=x^3$, where a is a variable parameter.
- Show that (4x+3y+1) dx + (3x+2y+1) dy = 0 represents hyperbolas having the following lines as asymptotes

  x+y = 0, 2x+y+1 = 0. **(1998)**
- Solve the following differential equation $y(1+xy)dx+x(1-xy)dy=0$.
- Solve the following differential equation $(D^2+4)y=\sin 2x$ given that when x = 0 then y = 0 and $\frac{dy}{dx}=2$.
- Solve $(D^3-1)y=xe^x+\cos^2 x$.
- Solve $(x^2D^2+xD-4)y=x^2$.

1991

- If the equation Mdx + Ndy = 0 is of the form $f_1(xy).ydx + f_2(xy).x\,dy = 0$, then show that $\frac{1}{Mx - Ny}$ is an integrating factor provided $Mx - Ny \neq 0$.
- Solve the differential equation. $(x^2 - 2x + 2y^2)\,dx + 2xy\,dy = 0$.
- Given that the differential equation $(2x^2y^2 + y)\,dx - (x^3y - 3x)\,dy = 0$ has an integrating factor of the form $x^h y^k$, find its general solution.
- Solve $\frac{d^4y}{dx^4} - m^4 y = \sin mx$.
- Solve the differential equation
  $$\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = e^x.$$
- Solve the differential equation
  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = xe^{-x}$, given that y = 0 and $\frac{dy}{dx} = 0$, when x = 0.

1990

- If the equation $\lambda^n + a_1\lambda^{n-1} + \dots\dots\dots + a_n = 0$ (in unknown λ) has distinct roots $\lambda_1, \lambda_2, \dots\dots\dots \lambda_n$. Show that the constant coefficients of differential equation
  $\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \dots\dots\dots + a_{n-1}\frac{dy}{dx} + a_n = b$ has the most general solution of the form
  $y = c_0(x) + c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} + \dots\dots\dots + c_n e^{\lambda_n x}$.
  where $c_1, c_2 \dots\dots c_n$ are parameters. what is $c_0(x)$?
- Analyse the situation where the λ – equation in (a) has repeated roots.
- Solve the differential equation
  $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ is explicit form. If your answer contains imaginary quantities, recast it in a form free of those.
- Show that if the function $\frac{1}{t - f(t)}$ can be integrated (w.r.t 't'), then one can solve $\frac{dy}{dx} = f(y/x)$, for any given *f*. Hence or otherwise.
  $$\frac{dy}{dx} + \frac{x - 3y + 2}{3x - y + 6} = 0$$
- Verify that $y = (\sin^{-1}x)^2$ is a solution of $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. Find also the most general solution.

1989

- Find the value of y which satisfies the equation
  $(xy^3 - y^3 - x^2e^x) + 3xy^2\frac{dy}{dx} = 0$; given that y=1 when x = 1.
- Prove that the differential equation of all parabolas lying in a plane is $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^{-2/3} = 0$.
- Solve the differential equation
  $$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2.$$

1988

- Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2e^x \sin x$.
- Show that the equation $(12x+7y+1)\,dx + (7x+4y+1)\,dy = 0$ represents a family of curves having as asymptotes the lines $3x+2y-1=0$, $2x+y+1=0$.
- Obtain the differential equation of all circles in a plane n the form $\frac{d^3y}{dx^3}\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} - 3\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)^2 = 0$.

1987

- Solve the equation $x\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} = y + e^x$
- If $f(t) = t^{p-1}$, $g(t) = t^{q-1}$ for t > 0 but f(t) = g(t) = 0 for $t \le 0$, and h(t) = f * g, the convolution of f, g show that $h(t) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} t^{p+q-1}; t \ge 0$ and p, q are

positive constants. Hence deduce the formula $B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

1985

- Consider the equation $y'+5y=2$. Find that solution ϕ of the equation which satisfies $\phi(1)=3\phi'(0)$.
- Use Laplace transform to solve the differential equation $x''-2x'+x=e^t, \left('=\frac{d}{dt}\right)$ such that $x(0)=2, x'(0)=-1$.
- For two functions f, g both absolutely integrable on $(-\infty,\infty)$, define the convolution f * g.

  If L(f), L(g) are the Laplace transforms of f, g show that L (f * g) = L(f). L(g).
- Find the Laplace transform of the function
  $$f(t)=\begin{cases}1 & 2n\pi \le t < (2n+1)\pi \\ -1 & (2n+1)\pi \le t \le (2n+2)\pi\end{cases}$$
  n = 0, 1, 2, ...

1984

- Solve $\frac{d^2y}{dx^2}+y=\sec x$.
- Using the transformation $y=\frac{u}{x^k}$, solve the equation $xy''+(1+2k)y'+xy=0$.
- Solve the equation $(D^2+1)x=t\cos 2t$, given that $x_0=x_1=0$ by the method of Laplace transform.

1983

- Solve $x\frac{d^2y}{dx}+(x-1)\frac{dy}{dx}-y=x^2$.
- Solve $(y^2+yz)\,dx+(xz+z^2)\,dy+(y^2-xy)\,dz=0$.
- Solve the equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=t$ by the method of Laplace transform, given that y = –3 when t = 0, y = –1 when t = 1.