IFoS
PREVIOUS YEARS QUESTIONS (2017-2000) SEGMENT-WISE

COMPLEX ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2017

- If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of z. **(8)**
- Prove by the method of contour integration that
$$\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 0.$$ **(10)**
- Find the sum of residues of $f(z) = \dfrac{\sin z}{\cos z}$ at its poles inside the circle $|z| = 2$. **[8]**

2016

- Find the analytic function of which the real part is $e^{-x}\left\{\left(x^2 - y^2\right)\cos y + 2xy\sin y\right\}$. **(8)**
- Find the Laurent series for the function $f(z) = \dfrac{1}{1-z^2}$ with centre $z = 1$. **(10)**
- Evaluate by Contour integration $\displaystyle\int_0^{\pi} \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$. **(10)**

2015

- Let $u(x, y) = \cos x \sinh y$. Find the harmonic conjugate $v(x, y)$ of u and express $u(x, y) + i\,v(x, y)$ as a function of $z = x + iy$. **(8)**
- Evaluate the integral $\displaystyle\int_r \frac{z^2}{(z^2+1)(z-1)^2} dz$, where r is the circle $|z| = 2$ **(12)**
- Show that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem. **(15)**

2014

- Using Cauchy integral formula, evaluate
$$\int_C \frac{z+2}{(z+1)^2(z-2)} dz$$
where C is the circle $|z-i| = 2$
- Find the constants *a*, *b*, *c* such that the function $f(z)=2x^2-2xy-y^2+i(ax^2-bxy+cy^2)$ is analytic for all $z(=x+iy)$ and express $f(z)$ in terms of z. **(8)**
- Evaluate :
$$\int_C \frac{z}{z^4-6z^2+1} dz$$
when C is the circle $|z - i| = 2$ **(8)**
- Find the bilinear transformation which map the points -1, ∞, i into the points–
  (i) $i, 1, 1 + i$
  (ii) $\infty, i, 1$
  (iii) $0, \infty, 1$ **(15)**
- Find the Laurent series expansion at z=0 for the function
$$f(z) = \frac{1}{z^2(z^2+2z-3)}$$
in the regions (i) $1<|z|<3$ and (ii) $|z| > 3$. **(15)**

2013

- Construct an analytic function $f(z) = u(x, y) + iv(x, y)$, where $v(x, y) = 6xy - 5x + 3$. Express the result as a function of z.
- Evaluate $\displaystyle\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$.
- Find Laurent series about the indicated singularity. Name the singularity and give the region of convergence.
$$\frac{z-\sin z}{z^3}; z = 0.$$

2012

- Evaluate the integral

$$\int_{2-i}^{4+i} (x+y^2-ixy)dz$$

along the line segment AB joining the points A(2,–1) and B(4, 1). **(10)**

- Show that the function $u(x,y)=e^{-x}(x\cos y+y\sin y)$ is harmonic. Find its conjugate harmonic function $v(x,y)$ and the corresponding analytic function f(z). **(13)**

- Using the Residue Theorem, evaluate the integral

$$\int_C \frac{e^z-1}{z(z-1)(z+i)^2}dz,$$

where C is the circle |z| = 2 **(13)**

2011

- Expand the function

$$f(z)=\frac{2z^2+11z}{(z+1)(z+4)}$$

in a Laurent's series valid for $2<|z|<3$. **(10)**

- Examine the convergence of

$$\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$$ and evaluate, if possible. **(10)**

- State Cauchy's residue theorem. Using it, evaluate the integral

$$\int_C \frac{e^{z/2}}{(z+2)(z^2-4)}dz$$

Counterclockwise around the circle C:|z+1|=4. **(13)**

2010

- Determine the analytic function

$f(z)=u+iv$ if $v=e^x(x\sin y+y\cos y)$ **(10)**

- Using the method of contour integration, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)^2(x^2+2x+2)}$$ **(14)**

- Obtain Laurent's series expansion of the function

$f(z)=\dfrac{1}{(z+1)(z+3)}$ in the region $0<|z+1|<2$

(13)

2009

- Evaluate

$$\int_C \frac{2z+1}{z^2+z}dz$$

By Cauchy's integral formula, where C is $|z|=\frac{1}{2}$

(10)

- Determine the analytic function $w=u+iv$, is

$$u=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$$ **(13)**

- Evaluate by contour integration

$$\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2},\ 0<a<1$$ **(13)**

2008

- Evalutate $\int_C \bar{z}\,dz$ from $z=0$ to $z=4+2i$. along the curve given by $z=t^2+it$. **(10)**

- Expand in a Laurent's series the function $f(z)=\dfrac{1}{(z-1)z^2}$ about $z=0$. **(13)**

- Find the residiue of $f(z)=\tan z$ at $\pi/2$. **(13)**

2007

- If $f(z)=u+iv$ is analytic and $u=e^{-x}(x\sin y-y\cos y)$ then find v and f(z). **(10)**

- Applying Cauchy's criterion for convergence, show that the sequence (S_n) defined by $S_n=1+\frac{1}{2}+\frac{1}{3}+....+$ is not convergent. **(13)**

- Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in a Laurent series valid for $(i)\ 1<|z|>3.$ $(ii)\ |z|>3.$ **(13)**

- Using residue theorem, evaluate

$$\int_0^{2\pi} \frac{d\theta}{(3-2\cos\theta+\sin\theta)}$$

2005

- If f is analytic, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|^2$$ **(10/2006)**

- Show that the transformation $w=\frac{5-4z}{4z-2}$ maps unit circle $|z|=1$ onto a circle of radius unity and centre at $-1/2$ **(13/2006)**

- Use contour integration technique to find the value of $\int_0^{2p}\frac{d\theta}{2+\cos\theta}$

(14/2006)

2004

- Investigate the continuity at $(0,0)$ of the function

$$f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2}(x,y)\neq(0,0)\\ 0 \qquad (x,y)=(0,0)\end{cases}$$ **(10)**

- Find the analytic function $f(z)=u(x,y)+iv(x,y)$ for which $u-ve^x(\cos y-\sin y)$.

- Find the bilinear transformation that maps $z=1,0,\infty$ to $w=0,-\infty,1$ respectively. **(13)**

- Find the singular points with their nature and the residues there at of $f(z)=\frac{\cot\pi z}{\left(z-1/3\right)^2}$ **(13)**

- Prove that a function analytic for all finite values of z and bounded, is a constant. **(13)**

2003

- If $w=f(z)=u(x,y)+iv(x,y), z=x+iy,$ is analytic in a domain, show that $\frac{\partial w}{\partial \bar{z}}=0$. Hence or otherwise, show that $\sin(x+i3y)$ cannot be analytic

- Discuss the transformation $w=z+\frac{1}{z}$ and hence, show that
  (1) a circle in z-plane is mapped on an ellipse in the w-plane
  (2) a line in the z-plane is mapped into a hyperbola in the w-plane. **(13)**

- Find the Laurent series expansion of the function $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ Valid in the region $2<|z|<3.$ **(13)**

2002

- If $f(z)$ has a simple pole with residue K at the origin and is analytic on $0<|z|\leq$ Show that

$$\frac{1}{2\pi i}\oint_C\frac{f(z)}{(z-a)(z-b)}dz=\frac{f(a)-f(b)}{a-b}+\frac{K}{ab}$$

Where $0<a, b<1$ *and* C is the circle $|z|=1$.

- If $f(a)=\oint_C\frac{3z^2+7z+1}{z-a}dz$ Where C is the circle $|z|=2$; Find
  (i) $f(1-i)$; (ii) $f''(1-i)$; (iii) $f(1+i)$ **(12)**

- Under the bilinear transformation $w=\frac{3-z}{z-2}$
  Find the images of
  (1) $\left|z-\frac{5}{2}\right|=1/2$ *and*
  (2) $\left|z-\frac{5}{2}\right|<1/2$ *in the W – plane.*

2001

- Compute the Taylor series around $z=0$ and give the radius of convergence for $\frac{z}{z-1}$

- Show that the function $f(z)=\sqrt{xy}$ is not regular at the origin although the Cauchy-Riemann equations are satisfied **(13)**

- By using the Residue Theorem evalutae the integral

$$\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2} \quad Where \quad 0<a<1.$$ **(14)**

2000

- Expand the function $f(z)=\log(z+2)$ in a power series and determine its radius of convergence.
- Prove that the function $f(z)=u+iv$

  Where $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$

  $f(0)=0$

  Satisfies Cauchy-Riemann equations at the origin, but $f'(0)$ does not exist.