IMS™ Institute of Mathematical Sciences

Institute for IAS/ IFoS/ CSIR/GATE Examinations

Previous Years Questions (1983–2011) Segment-wise

Vector Analysis Paper – I

(According to the New Syllabus Pattern)

1983

- Prove that curl (curl F) = grad div F - $\nabla^2 F$.

1985

- If P,Q,R are points (3,-2,-1) , (1,3,4) , (2,1,-2) respectively. Find the distance from P to the plane OQR, where 'O' is the origin.
- Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} - 2t\,\hat{j} + t^3\hat{k}$ at the points t=1 and t=2
- Find div F and curl F, where $F = \nabla(x^3 + y^3 + z^3 - 3xyz)$

1986

- Let $\vec{a}, \vec{b}$ be given vectors in the three dimensional Euclidean space E_3 and let $\phi(x)$ be a scalar field of the vectors x also of E_3 . If $\phi(x) = (\vec{x}\times\vec{a}).(\vec{x}\times\vec{b})$, show that grad $\phi(i.e, \nabla\phi(x)) = \vec{b}\times(\vec{x}\times\vec{a}) + \vec{a}\times(\vec{x}\times\vec{b})$.
- If $\vec{f}, \vec{g}$ are two vector fields in E_3 and if 'div', 'curl' are defined on an open set $S \subset E_3$ show that $div(\vec{f}\times\vec{g}) = \vec{g}.\,curl\,\vec{f} - \vec{f}.curl\,\vec{g}$. ***(1988)***

1987

- Show that for a vector field $\vec{f}$, curl (curl $\vec{f}$) = grad (div $\vec{f}$) - $\nabla^2\vec{f}$.
- If $\vec{r}$ is the position vector to a point whose distance from the origin is r , prove that $div\,\vec{f} = 0$ if $\vec{f} = \frac{\vec{r}}{r^3}$.
- Prove that for a three vectors $\vec{a}, \vec{b}, \vec{c}$ $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{a}\left(\vec{b}.\vec{c}\right) - \vec{c}\left(\vec{a}.\vec{b}\right)$ and explain its geometric meaning. ***(1990)***

1988

- Define the divergence of a vector point function , prove that $div\,(\vec{u}\times\vec{v}) = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$. ***(1986)***
- Using Gauss divergence theorem, evaluate $\iint_S (x\hat{i} + y\hat{i} + z^2\hat{k}).\hat{n}\,ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = 2$ and the plane Z=1 and $\hat{n}$ is the outward unit normal to S .

1989

- Define the curl of a vector point function
- Prove that $\nabla\times(\frac{\vec{r}}{r^2}) = 0$ where $\vec{r} = (x, y, z)$ and $r = |\vec{r}|$.

1991

- If ϕ be a scalar point function and F be a vector point function, show that the components of F normal and tangential to surface $\phi = 0$ at any point there of are $\frac{(F.\nabla\phi)\nabla\phi}{(\nabla\phi)^2}$ and $\frac{\nabla\phi\times(F\times\nabla\phi)}{(\nabla\phi)^2}$

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- Find the value of $\int$ curl F. ds taken over the portion of the surface $x^2+y^2-2ax+az=0$, for which $Z \geq 0$, when $F=(y^2+z^2-x^2)\,\hat{i}+(z^2+x^2-y^2)\,\hat{j}+(x^2+y^2-z^2)\hat{k}$.

1992

- If $\vec{f}(x,y,z)=(y^2+z^2)\,\hat{i}+(z^2+x^2)\,\hat{j}+(x^2+y^2)\hat{k}$ then calculate $\int_c \vec{f}.d\vec{x}$ where 'C' consists of the line segment from (0,0,0) to (1,1,1)
- The three line segments AB,BC and CD, where A,B,C and D are respectively the points (0,0,0) , (1,0,0), (1,1,0) and (1,1,1)
- The curve $\vec{x}=u\hat{i}+u^2\hat{j}+u^3\hat{k}$, u from 0 to 1.
- If $\vec{a}$ and $\vec{b}$ are constant vectors , show that $(i)\ div\left\{\vec{x}\times(\vec{a}\times\vec{x})\right\}=-2\vec{x}.\vec{a}$

  $(ii)\ div\left\{(\vec{a}\times\vec{x})\times(\vec{b}\times\vec{x})\right\}=2\vec{a}.(\vec{b}\times\vec{x})-2\vec{b}.(\vec{a}\times\vec{x})$

1993

- Evaluate $\iint_s \nabla\times\vec{F}.\hat{n}\,ds$, where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k}$.

1994

- If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$. evaluate $\iint_s(\nabla\times\vec{F}).\hat{n}ds$.

1996

- If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$, show that

  (i) $\vec{r}\times grad\ f(r)=0$

  (ii) $div\ (r^n\vec{r})=(n+3)r^n$
- Verify Gauss divergence theorem for

  $\vec{F}=xy\hat{i}+z^2\hat{j}+2yz\hat{k}$, on the tetrahedron x = y = z = 0, x +y + z=1

1997

- Prove that if $\vec{A},\vec{B}$ and $\vec{C}$ are three given non coplanar vectors, then any vector $\vec{F}$ can be put in the form $\vec{F}=\alpha\vec{B}\times\vec{C}+\beta\vec{C}\times\vec{A}+\gamma\vec{A}\times\vec{B}$. For a given $\vec{F}$ determine α ,β ,γ.

1998

- If r_1 and r_2 are the vectors joining the fixed points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ respectively to a variable point P (x , y , z), then find the values of grad $(r_1 \cdot r_2)$ and curl $(r_1 \times r_2)$
- Show that $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$ if either $\vec{b}$ =0 (or any other vector is '0') or $\vec{c}$ is collinear with $\vec{a}$ or $\vec{b}$ is orthogonal to $\vec{a}$ and $\vec{c}$ (both).

1999

- If $\vec{a},\vec{b},\vec{c}$ are the position vectors A ,B, C prove that $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane ABC.

- If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find $\nabla \times \vec{F}$.
- Evaluate $\int_c (e^{-x} \sin y\,dx + e^{-x} \cos y\,dy)$; (by Green's theorem), where 'C' is the rectangle whose vertices are $(0,0)$, $(\pi,0)$ $(\pi,\pi/2)\&(0,\pi/2)$.
- If x , y, z are the components of a contra variant vector in rectangular cartesian co-ordinates x,y,z in a three dimensional space, show that the components of the vector in cylindrical co- ordinates $r,\theta,z\,are\; X\cos\theta + Y\sin\theta, \frac{-X}{r}\sin\theta + \frac{Y}{r}\cos\theta, Z$

2000

- In what direction from the point (-1 , 1 , 1) is the directional derivative of $f = x^2 y z^3$ a maximum ? compute its magnitude .
- Show that
  (i). $(A+B).(B+C)\times(C+A) = 2A.B\times C$
  (ii). $\nabla\times(A\times B) = (B.\nabla)A - B(\nabla.A) - (A.\nabla)B + A(\nabla.B)$ ***(1990)***
- Evaluate $\iint_S F.\hat{n}ds$ where $F = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the surface of the parallelopiped bounded by $x=0, y=0, z=0, x=2, y=1$ *and* $z=3$.

2001

- Find the length of the arc of the twisted curve $\vec{r} = (3t, 3t^2, 2t^3)$ from the point t = 0 to the point t = 1. Find also the unit tangent 't', unit normal 'n' and the unit binormal b at t = 1.
- Show that curl $\frac{\vec{a}\times\vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a}.\vec{r})$ where $\vec{a}$ *is a* constant vector
- Find the directional derivative of $f = x^2 y z^3$ along $x = e^{-t}, y = 1 + 2\sin t, z = t - \cos t$ at t = 0.
- Show that the vector field defined by $F = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find also the scalar 'u' such that F = grad u.
- Verify Gauss divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4$ z =0 & z = 3.

2002

- Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$. show that $\vec{R}\times\frac{d\vec{R}}{dt} = \frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ where $r = |\vec{r}|$.
- Find the curvature K for the space curve $x = a\cos\theta$, $y = a\sin\theta$, $z = a\theta\tan\alpha$
- Show that curl (curl $\vec{v}$) = grad ($div\vec{v}$) - $\nabla^2\vec{v}$
- Let D be a closed and bounded region having boundary S. Further let 'f' be a scalar function having second order partial derivatives defined on it. show that $\iint_S (f\,grad\,f).\hat{n}ds = \iiint_V \left[|grad|^2 + f\nabla^2 f\right]dv$ Hence or otherwise evaluate $\iint_S (f\,grad\,f).\hat{n}ds$ for f = 2x + y +2z over $S \equiv x^2 + y^2 + z^2 = 4$
- Find the values of constants a, b, and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at (1, -1, 1) is in the direction parallel to y axis and has magnitude 6.

2003

- Show that if $\vec{a}', \vec{b}'$ *and* $\vec{c}'$ are the reciprocals of the non – coplanar vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, then any vector $\vec{r}$ may be expressed as
  $\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')b + (\vec{r}.\vec{c}')c.$

- Prove that the divergence of a vector field is invariant w. r. t co – ordinate transformations.
- Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.
- Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A \times (\nabla \times A)$
  Where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.
- Find the radii of curvature and torsion at a point of intersection of the surfaces $x^2 - y^2 = c^2$, $y = x \tanh(\frac{z}{c})$
- Evaluate $\iint_S curl \ A.ds$ where S is the open surface
  $x^2 + y^2 - 4x + 4z = 0$, $z \geq 0$ and $A = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$

2004

- Show that if $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A} \times \vec{B}$ is solenoidal .
- Show that the Frenet – Serret formulae can be written in the form $\frac{d\vec{T}}{ds} = \vec{w} \times \vec{T}$, $\frac{d\vec{N}}{ds} = \vec{W} \times \vec{N}$ and $\frac{d\vec{B}}{ds}\vec{W} \times \vec{B}$
  Where, $\vec{W} = \tau \vec{T} + k\vec{B}$
- Prove the identity
  $\nabla \ (\vec{A} \cdot \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} + (\vec{A} \cdot \nabla)\vec{B} + \vec{B} \times (\nabla \times \vec{A}) + \vec{A} \times (\nabla \times \vec{B})$
- Derive the identity
  $\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv = \iint_S (\phi \nabla \psi - \psi \nabla \phi).\hat{n}\, ds$, where V is the volume bounded by the closed surface S.
- Verify Stoke's theorem for $\vec{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2005

- Show that the volume of the tetrahedron ABCD is $\frac{1}{6}(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}$. Hence find the volume of the tetrahedron with vertices (2, 2, 2) , (2, 0, 0) , (0, 2, 0) and (0, 0, 2).
- Prove that the curl of a vector field is independent of the choice of co – ordinates.
- The parametric equation of a circular helix is $\vec{r} = a\cos u\,\hat{i} + a\sin u\,\hat{j} + cu\,\hat{k}$; where 'c' is a constant and 'u' is a parameter.
- Find the unit tangent vector $\hat{t}$ at the point 'u' and the arc length measured from u = 0. Also find $\frac{d\hat{t}}{ds}$, where 'S' is the arc length.
- Show that $curl\left(\hat{K} \times grad\frac{1}{r}\right) + grad\left(\hat{K} \cdot grad\frac{1}{r}\right) = 0$ where r is the distance from the origin and $\hat{K}$ is the unit vector in the direction oz.
- Find the curvature and the torsion of the space curve $x = a(3u - u^3), y = 3au^2, z = a(3u + u^3)$.
- Evaluate $\iint_S (x^3 dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy)$ by Gauss divergence theorem, where S is the surface of the cylinde $x^2 + y^2 = a^2$ bounded by z =0 and z = b.

2006

- Find the values of constant a, b, and c so that the directional of the function $f = axy^2 + byz + cz^2x^3$ at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to Z – axis.
- If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} - 7\hat{k}$, determine a vector $\vec{R}$ satisfying the vector equations

$\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}\cdot\vec{A}=0$

- Prove that $r^n\vec{r}$ is an irrotational vector for any value of n , but is solenoidal only if n + 3 = 0 .
- If the unit tangent vector $\vec{t}$ and binormal $\vec{b}$ makes angles θ and ϕ respectively with a constant unit vector $\vec{a}$, prove that $\frac{Sin\theta}{Sin\phi}\cdot\frac{d\theta}{d\phi}=-\frac{k}{\tau}$
- Verify Stoke's theorem for the function $\vec{F}=x^2\hat{i}-xy\hat{j}$ integrated round the square in the plane z = 0 and bounded by the lines x=0, y = 0 , x =a and y = a , a > 0.

2007

- If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r=|\vec{r}|$ determine grad (r^{-1}) in terms of $\hat{r}$ and r.
- Find the curvature and torsion at any point of the curve
  $x=a\cos 2t$, $y=a\sin 2t$, $Z=2a\sin t$
- For any constant vector $\vec{a}$ show that the vector represented by curl $(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a}, \vec{r}$ being the position vector of a point (x, y, z), measured from the origin.
- If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ find the value(s) of n in order that $r^n\vec{r}$ may be (i) solenoidal or (ii) irrotational
- Determine
  $\int_C (y\,dx+z\,dy+x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by
  $(x-a)^2+(y-a)^2+z^2=2a^2$, $x+y=2a$ that starts from the point (2a, 0, 0) and goes at first below the z – plane.

2008

- Find the constants 'a' and 'b' so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1, -1, 2)
- Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).
  $P.T\ \nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=(x^2+y^2+z^2)^{1/2}$. Hence find f(r) such that $\nabla^2 f(r)=0$.
- Show that for the space curve $x=t,\ y=t^2,\ z=\frac{2}{3}t^3$ the curvature and torsion are same at every point.
- Evaluate $\int_c \vec{A}d\vec{r}$ along the curve $x^2+y^2=1,\ z=1$ from (0, 1, 1) to (1, 0,1) if $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$.
- Evaluate $\iint_s \vec{F}\cdot\hat{n}\,ds$ where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and 'S' is the surface of the cylinder bounded by $x^2+y^2=4$, z = 0 and z = 3.

2009

- Show that $div(grad\, r^n)=n(n+1)r^{n-2}$
  Where $r=\sqrt{x^2+y^2+z^2}$. **(12)**
- Find the directional derivatives of –
  (i) $4xz^3-3x^2y^2z^2$ at (2, -1, 2) along z – axis;
  (ii) $x^2yz+4xz^2$ at (1, -2, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$. **(12)**
- Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1,\ z=0$ under the field of force given by
  $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. **(20)**
- Using divergence theorem, evaluate

$\iint_S \vec{A}.d\vec{S}$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(20)**

- Find the value of $\iint_S (\vec{\nabla} \times \vec{F}).\,d\vec{S}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane z = 0, when $\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$. **(20)**

2010

- Find the directional derivative of $f(x,y) = x^2y^3 + xy$ at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x – axis. **(12)**
- Show that the vector field defined by the vector function $\vec{V} = xyz(yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k})$ is conservative. **(12)**
- Prove that $div(f\,\vec{v}) = f\,(div\,\vec{v}) + (grad\ f)\vec{v}$ where f is a scalar function. **(20)**
- Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}\,dA$ where $\vec{V} = x^2z\vec{i} + y\,\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane z = 4y. **(20)**
- Verify Green's theorem for ;
  $e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ the path of integration being the boundary of the square whose vertices are (0, 0), ($\pi/2$, 0), ($\pi/2$, $\pi/2$) and (0, $\pi/2$). **(20)**

2011

- For two vectors $\vec{a}$ *and* $\vec{b}$ given respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ *and* $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$
  Determine: $(i)\frac{d}{dt}(\vec{a}\cdot\vec{b})$ *and* $(ii)\frac{d}{dt}(\vec{a}\times\vec{b})$ **(10)**
- If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad\,v$, find the value of $\vec{f}\cdot curl\,\vec{f}$ **(10)**
- Examine whether the vectors $\nabla u, \nabla v$ *and* ∇w are coplanar, where u, v and w are the scalar functions defined by: u=x +y + z, $v = x^2 + y^2 + z^2$ and w= yz + zx + xy. **(15)**
- If $\vec{u} = 4y\hat{i} + x\hat{j} - 2z\hat{k}$, calculate the double integral $\iint(\nabla\times\vec{u})\cdot d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \ge 0$. **(15)**
- If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal. **(15)**
- Verify Gauss Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$ taken over the cube $0 \le x, y, z \le 1$. **(15)**

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