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136/250

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

TEST SERIES (MAIN)-2014

Test Code: FULL TEST P-II(M) IAS/T-08

MATHEMATICS

by K. VENKANNA

The person with 14 years of Teaching Experience

FULL TEST P-II

Time: Three Hours *Maximum Marks: 250*

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name: Varun Guntupalli

Roll No.

Test Centre: Hyderabad

Medium: English

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED	
1	(a)			–	
	(b)			–	
	(c)			–	
	(d)			08	16
	(e)			08	
2	(a)				
	(b)				
	(c)				
	(d)				
3	(a)				
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	(c)				
	(d)				
4	(a)			–	
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6	(a)			10	
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8	(a)				
	(b)				
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	(d)				
			Total Marks	136/250	

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THIS SPACE

1. (c) Examine the convergence of $\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x}$. (10)

$$I = \int_0^\infty \frac{1}{x}\left(\frac{1}{1+x}\right)dx - \int_0^\infty \frac{e^{-x}}{x}\,dx$$

1. (d) If $u+v=\dfrac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, and $f(z)=u+iv$ is an analytic function of $z=x+iy$, find $f(z)$ in terms of z. **[10]**

$$\Rightarrow u_x+v_x = u_x-u_y = \frac{4\cos 2x\,(e^{2y}+e^{-2y}-2\cos 2x)-8\sin^2 2x}{(e^{2y}+e^{-2y}-2\cos 2x)^2}$$

$$= \frac{4\cos 2x\,(e^{2y}+e^{-2y})-8}{(e^{2y}+e^{-2y}-2\cos 2x)^2} \quad -(1)$$

$$u_y+v_y = u_y+u_x = \frac{-2\sin 2x\,(2e^{2y}-2e^{-2y})}{(e^{2y}+e^{-2y}-2\cos 2x)^2}$$

$$= \frac{-4\sin 2x\,(e^{2y}-e^{-2y})}{(e^{2y}+e^{-2y}-2\cos 2x)^2} \quad -(2)$$

$$\Rightarrow u_x = \frac{e^{2y}(2\cos 2x-2\sin 2x)+e^{-2y}(2\cos 2x+2\sin 2x)-4}{(e^{2y}+e^{-2y}-2\cos 2x)^2} \quad -(3)$$

$$u_y = \frac{-e^{2y}(2\sin 2x+2\cos 2x)+e^{-2y}(2\sin 2x-2\cos 2x)+4}{(e^{2y}+e^{-2y}-2\cos 2x)^2} \quad -(4)$$

$$\Rightarrow f(z) = \int \phi_1(z,0) - i\int \phi_2(z,0) \quad \text{where } \phi_1 = u_x,\ \phi_2 = u_y$$

$$= \int \frac{4\cos 2z-4}{(2-2\cos 2z)^2}\,dz - i\int \frac{-4\cos 2z+4}{(2-2\cos 2z)^2}\,dz$$

$$= \int \frac{1}{\cos 2z-1}\,dz - i\int \frac{1}{1-\cos 2z}\,dz$$

$$= (1+i)\int -\operatorname{cosec}^2 z\,dz = (1+i)(\cot z + c)$$

$$= (1+i)\cot z + c_1 \quad \text{where } c_1 \text{ is an arbitrary constant.}$$

1. (e) There are five pumps available for developing five wells. The efficiency of each pump in producing the maximum yield in each well is shown in the table below. In what way should the pumps be assigned so as to maximise the overall efficiency?

Efficiency Well

		W_1	W_2	W_3	W_4	W_5
	P_1	45	40	65	30	55
	P_2	50	30	25	60	30
Pump	P_3	25	20	15	20	40
	P_4	35	25	30	25	20
	P_5	80	60	60	70	50

[10]

Converting this table into minimization assignment problem, take −ves for each value

	W_1	W_2	W_3	W_4	W_5
P_1	−45	−40	−65	−30	−55
P_2	−50	−30	−25	−60	−30
P_3	−25	−20	−15	−20	−40
P_4	−35	−25	−30	−25	−20
P_5	−80	−60	−60	−70	−50

Subtracting least cost element from each row, ~~& thereafter from each column~~

	W_1	W_2	W_3	W_4	W_5
P_1	20	25	0	35	10
P_2	10	30	35	0	30
P_3	15	20	25	20	0
P_4	0	10	5	10	15
P_5	0	20	20	10	30

Now, subtracting least cost element from each column,

	W_1	W_2	W_3	W_4	W_5
P_1	20	15	$\boxed{0}$	35	10
P_2	10	20	35	$\boxed{0}$	30
P_3	15	10	25	20	$\boxed{0}$
P_4	0	$\boxed{0}$	5	10	15
P_5	$\boxed{0}$	10	20	10	30

Least possible lines covering all zeros = 5

⇒ This gives optimal solution.

⇒ optimal assignment for maximizing efficiency

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$= P_1 \to W_3,\ P_2 \to W_4,\ P_3 \to W_5,$
$P_4 \to W_2,\ P_5 \to W_1$

2. (a) Let $\mathbf{R}^*$ be the group of nonzero real numbers under multiplication and let $H = \{x \in \mathbf{R}^* \mid x^2$ is rational$\}$. Prove that H is a subgroup of $\mathbf{R}^*$. Can the exponent 2 be replaced by any positive integer and still have H be a subgroup?

[10]

4. (a) Find an integer n that shows that the rings $\mathbb{Z}_n$ need not have the following properties that the ring of integers has.

(i) $a^2 = a$ implies $a = 0$ or $a = 1$.

(ii) $ab = 0$ implies $a = 0$ or $b = 0$.

(iii) $ab = ac$ and $a \neq 0$ imply $b = c$.

Is the n you found prime? **(12)**

4. (b) Let $f: \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable on $\mathbf{R}$ but f' is not continuous on $\mathbf{R}$. (13)

Clearly, f is a continuous function for all $x > 0$ & $x < 0$

Hence, $f(x)$ is differentiable on $R - \{0\}$

At $x = 0$,

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^2 \sin 1/h^2 - 0}{h}$$

$$= \lim_{h\to 0} h \sin 1/h^2$$

$$= 0 \quad \left[\because \sin\frac{1}{h^2} \in [-1, 1] \text{ while } h \to 0 \right]$$

$$\Rightarrow f'(0) = 0$$

i.e. f is differentiable on R

$$f'(x) = 2x \sin\frac{1}{x^2} + x^2 \cos\frac{1}{x^2} \times \left(-\frac{2}{x^3}\right)$$

$$= 2x \sin\frac{1}{x^2} - \frac{2}{x}\cos\frac{1}{x^2}$$

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0}\left\{2x\sin\frac{1}{x^2} - \frac{2}{x}\cos\frac{1}{x^2}\right\}$$

which does not exist

$\Rightarrow f'$ is not continuous on R.

4. (c) Expand $f(z)=\frac{z+3}{z(z^2-z-2)}$ in powers of z; where (i) $|z|<1$, (ii) $1<|z|<2$, (iii) $|z|>2$. (12)

$$f(z) = \frac{-3/2}{z} + \frac{5/6}{z-2} + \frac{2/3}{z+1} \quad -(1)$$

(i) $|z|<1$

$$\Rightarrow f(z) = -\frac{3}{2z} + \frac{5}{6}\cdot -\frac{1}{2}\left(1-\frac{z}{2}\right)^{-1} + \frac{2}{3}\cdot(1+z)^{-1}$$

$$= -\frac{3}{2z} - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n + \frac{2}{3}\sum_{n=0}^{\infty}(-1)^n z^n$$

(ii) $1<|z|<2$

$$\Rightarrow f(z) = -\frac{3}{2z} + \frac{5}{6}\cdot\frac{-1}{2}\left(1-\frac{z}{2}\right)^{-1} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1}$$

$$= -\frac{3}{2z} - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n + \frac{2}{3z}\sum_{n=0}^{\infty}(-1)^n\frac{1}{z^n}$$

(iii) $|z|>2$

$$\Rightarrow f(z) = -\frac{3}{2z} + \frac{5}{6}\cdot\frac{1}{z}\left(1-\frac{2}{z}\right)^{-1} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1}$$

$$= -\frac{3}{2z} + \frac{5}{6z}\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^n + \frac{2}{3z}\sum_{n=0}^{\infty}(-1)^n\frac{1}{z^n}$$

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4. (d) Make a graphical representation of the set of constraints of the following LPP. Find the extreme points of the feasible region. Finally, solve the problem graphically.

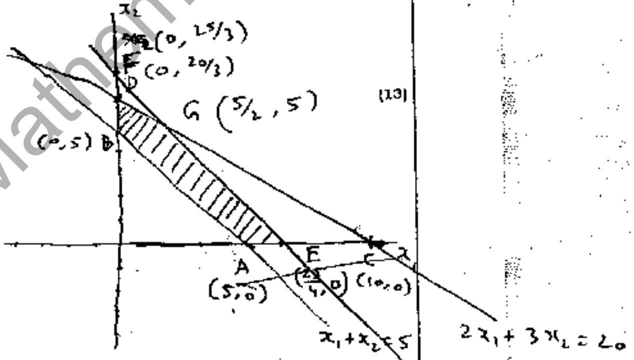
Maximise $Z = 2x_1 + x_2$

subject to $x_1 + x_2 \geq 5$

$2x_1 + 3x_2 \leq 20$

$4x_1 + 3x_2 \leq 25$

$x_1, x_2 \geq 0.$ [13]



Feasible region is AEGDB & extreme pts are $A(5,0)$, $E(\frac{25}{4}, 0)$, $G(5/2, 5)$, $D(0, 20/3)$, $B(0,5)$

Given optimization problem is

Maximize $Z = 2x_1 + x_2$

At $A(5,0)$, $Z = 10$

$E\left(\frac{25}{4}, 0\right)$, $Z = \frac{25}{2}$ ——— maximum value.

$G\left(5/2, 5\right)$, $Z = 10$

$D\left(0, 20/3\right)$, $Z = 20/3$

$B(0,5)$, $Z = 5$

$\Rightarrow$ Max $Z = 10$ at

$x_1 = 5,\ x_2 = 0$ or $x_1 = 5/2,\ x_2 = 5$

SECTION-B

5. (a) Solve $x^2p^2 + y^2q^2 = z^2$. [10]

$$x^2p^2 + y^2q^2 = z^2 \quad -(1)$$

$$\Rightarrow \left(\frac{\frac{1}{z}\partial z}{\frac{1}{x}\partial x}\right)^2 + \left(\frac{\frac{1}{z}\partial z}{\frac{1}{y}\partial y}\right)^2 = 1$$

Put $\frac{dx}{x} = dX$, $\frac{dy}{y} = dY$, $\frac{dz}{z} = dZ$.

$\Rightarrow \ln x = X$, $\ln y = Y$, $\ln z = Z$

$\Rightarrow P^2 + Q^2 = 1 \quad -(2)$ where $\frac{\partial Z}{\partial X} = P$ & $\frac{\partial Z}{\partial Y} = Q$

$\Rightarrow$ Solution of (2) is

$Z = aX + bY + c$ where $a^2 + b^2 = 1$

$\Rightarrow Z = aX + \sqrt{1-a^2}\,Y + c$

$\Rightarrow \ln z = a\ln x + \sqrt{1-a^2}\,\ln y + c$

$\Rightarrow z = c_1 x^a y^{\sqrt{1-a^2}}$

where a, c_1 are arbitrary constants.

5. (b) Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$. (10)

$$(D - D')^2 z = 6$$

$$\Rightarrow CF = \phi_1(y+x) + x\phi_2(y+x) \quad \text{--- (1)}$$

$$PI = \frac{1}{(D-D')^2} 6 = 3x^2 \quad \text{--- (2)}$$

$$\Rightarrow z = \phi_1(y+x) + x\phi_2(y+x) + 3x^2$$

$$\Rightarrow \frac{\partial z}{\partial x} = \phi_1'(y+x) + x\phi_2'(y+x) + \phi_2(y+x) + 6x$$

$$\frac{\partial z}{\partial y} = \phi_1'(y+x) + x\phi_2'(y+x)$$

At $x=y$, $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = x$

$\Rightarrow x = \phi_1'(2x) + x\phi_2'(2x) + \phi_2(2x) + 6x$

$x = \phi_1'(2x) + x\phi_2'(2x)$

$\Rightarrow \phi_2(2x) = -6x \Rightarrow \phi_2(x) = -3x$

$\Rightarrow \phi_2'(2x) = -3 \Rightarrow \phi_1'(2x) = 4x$

$\Rightarrow \phi_1'(x) = 2x \Rightarrow \phi_1(x) = x^2 + c$

$\Rightarrow z = (y+x)^2 + c + x(-3(x+y)) + 3x^2$

$= (x+y)^2 + c - 3xy$

At $x=y$, $z = xy = x^2 = (x+x)^2 + c - 3x^2$

$= x^2 + c$

$\Rightarrow c = 0$

$\Rightarrow z = (x+y)^2 - 3xy$

$\Rightarrow z = x^2 - xy + y^2$

5. (c) The current i in an electric circuit is given by
$i = 10e^{-t}\sin 2\pi t$ where t is in seconds. Using Newton's method, find the value of t correct to 3 decimal places for $i = 2$ amp. (10)

~~For $i=2$, $f(t) = 5\sin 2\pi t - 1 = 0$~~

Newton Raphson method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $i=2$, $f(t) = 5\sin 2\pi t - e^{t} = 0$

$$f'(t) = 10\pi\cos 2\pi t - e^{t}$$

Put $x_0 = 0$

$$\Rightarrow x_1 = 0 - \frac{f(0)}{f'(0)} = \frac{1}{30.416} = 0.03287$$

$$\cancel{x_1 = 0.033 - \frac{f(0.033)}{f'(0.033)} = 0.033 - \frac{-1.0154}{29.709} = 0.066}$$

$$x_2 = 0.066 - \frac{-1.032}{30.3568} = 0.1$$

$$x_1 = 0.03287 - \frac{(-0.008)}{29.7149} = 0.03314$$

$$\cancel{x_2 = 0.034 - \frac{f(0.034)}{f'(0.034)} = 0.034 - \frac{-0.0254}{29.667} =}$$

$$x_2 = 0.03314 - \frac{f(0.03314)}{f'(0.03314)} = 0.03314 - \frac{-0.000078}{29.7036}$$

$$= 0.03314$$

$\Rightarrow$ Value of t correct to 3 decimal places

$= 0.033$

5. (d) (i) Realize the following expression by using NAND gates only.

$g = (\bar{a} + \bar{b} + c)\bar{d}\,(\bar{a} + e)f$

where $\bar{x}$ denotes the complement of x.

(ii) Find the decimal equivalent of $(357.32)_8$ [10]

(i)

(ii) $(357.32)_8 = 3\times 8^2 + 5\times 8^1 + 7\times 8^0 + 3\times 8^{-1} + 2\times 8^{-2}$

$= 3\times 64 + 5\times 8 + 7 + \frac{3}{8} + \frac{2}{64}$

$= 192 + 40 + 7 + \frac{13}{32}$

$= 239.40625$

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5. (c) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M', starting from the upper end, walks down the plank so that it does not move, show that he gets to the other end in time

$$\sqrt{\left\{\frac{2M'a}{(M+M')g\sin\alpha}\right\}}$$, where a is the length of the plane. (10)

6. (a) Solve $(D+D'-1)(D+D'-3)(D+D')z = e^{x+y}\sin(2x+y)$ — (1) [12]

$$CF = e^{x}\phi_1(y-x) + e^{3x}\phi_2(y-x) + \phi_3(y-x)$$

$$PI = \frac{1}{(D+D'-1)(D+D'-3)(D+D')}\, e^{x+y}\sin(2x+y)$$

$$= e^{x+y}\frac{1}{(D+D'+1)(D+D'-1)(D+D'+2)}\sin(2x+y)$$

$$= e^{x+y}\frac{1}{(D^2+2DD'+D'^2-1)(D+D'+2)}\sin(2x+y)$$

$$= e^{x+y}\frac{(D+D'-2)}{(-4+2\times(-2)-1-1)((D+D')^2-4)}\sin(2x+y)$$

$$= e^{x+y}\frac{D+D'-2}{-10\times(-4-4-1-4)}\sin(2x+y)$$

$$= e^{x+y}\cdot\frac{1}{130}\times\big(2\cos(2x+y)+\cos(2x+y)-2\sin(2x+y)\big)$$

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$$= \frac{1}{130}e^{x+y}\big(3\cos(2x+y)-2\sin(2x+y)\big)$$

$\Rightarrow$ Solution of (1) is $Z = CF + PI = e^{x}\phi_1(y-x) + e^{3x}\phi_2(y-x) + \phi_3(y-x) + \frac{1}{130}e^{x+y}\left[3\cos(2x+y) - 2\sin(2x+y)\right]$.

6. (b) Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$. (13)

$$\left(D^2 - 2DD' + D'^2\right) z = 6 \quad - (1)$$

Auxiliary equation is $(m-1)^2 = 0 \Rightarrow m = 1, 1$

$$\Rightarrow CF = \phi_1(y+x) + x\,\phi_2(y+x) \quad - (2)$$

$$PI = \frac{1}{(D-D')^2}\,6 \;\cancel{= \frac{1}{D^2}\,6}$$

$$= 3x^2$$

$$\Rightarrow z = \phi_1(y+x) + x\,\phi_2(y+x) + 3x^2 \quad - (3)$$

(3) touches $z = xy$ along $y = x$

$\Rightarrow \frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ are same for (3) & $z = xy$

at $x = y$

From (3), $\frac{\partial z}{\partial x} = \phi_1'(y+x) + x\phi_2'(y+x) + \phi_2(y+x) + 6x$ — (4)

$\frac{\partial z}{\partial y} = \phi_1'(y+x) + x\phi_2'(y+x)$ — (5)

For $z = xy$, $\frac{\partial z}{\partial x} = y$; $\frac{\partial z}{\partial y} = x$ — (6)

$\Rightarrow$ From (4), (5) & (6)

$x = \phi_1'(2x) + x\phi_2'(2x) + \phi_2(2x) + 6x$

& $x = \phi_1'(2x) + x\phi_2'(2x)$

$\Rightarrow \phi_2(2x) = -6x \Rightarrow \underline{\phi_2(x) = -3x}$

$\Rightarrow \phi_2'(2x) = -3 \Rightarrow \phi_1'(2x) = 4x$

$\cancel{\Rightarrow \phi_1(2x) = \frac{4x^2}{2} \Rightarrow \phi_1(x) = \frac{7}{8}x^2}$

$\cancel{\Rightarrow z = \frac{7(x+y)^2}{2} \Rightarrow z = \frac{7}{8}(x+y)^2 + x[-3(x+y)]}$

$\phi_1'(x) = 2x$ $\cancel{\Rightarrow 3x^2}$

$\Rightarrow \phi_1(x) = x^2 + c_1$

$\cancel{\text{At } x = y,\ z = xy = x^2}$

At $x = y$, $x^2 = z \Rightarrow c_1 = 0$

$\cancel{\Rightarrow x^2 = \frac{7}{8}(2x)^2 + c_1 - 6x^2 + 3x^2}$

$\cancel{\Rightarrow z = \frac{7}{8}(x+y)^2 - 3xy}$ $\Rightarrow \underline{z = (x+y)^2 - 3xy}$

6. (c) The following table gives the velocity v of a particle at time t :

t (seconds) :	0	2	4	6	8	10	12
v (m/sec) :	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds and also the acceleration at t=2 sec. **(10)**

Using simpson's $\frac{1}{3}$ rule

$$I = \int_0^{12} v\,dt = \frac{1}{3}h\left[v_0 + v_6 + 4(v_1+v_3+v_5) + 2(v_2+v_4)\right]$$

$$= \frac{2}{3}\left[4 + 136 + 4\times(6+34+94) + 2\times(16+60)\right]$$

$$= 552 \text{ m.}$$

$\Rightarrow$ Distance moved in 12 seconds $= \underline{552 \text{ m}}$

~~Acceleration~~

$$\text{Distance in 2 seconds} = \int_0^2 v\,dt = \frac{h}{2}[v_0+v_1]$$

$$= \frac{2}{2}[4+6] = 10$$

(by trapezoidal rule)

~~Equation~~ $v^2 - u^2 = 2as$

$$\Rightarrow 36 - 16 = 2\times a\cdot 10$$

$$\Rightarrow \underline{a = 1 \text{ m/s}^2}$$

$\Rightarrow$ Acceleration at $t = 2$ sec

$= \underline{1 \text{ m/s}^2}$

6. (d) Solve the equations $27x+6y-z=85$; $x+y+54z=110$; $6x+15y+2z=72$ by Gauss-Seidal method. [15]

$$x = \frac{1}{27}(85-6y+z) \quad -(1)$$

$$y = \frac{1}{15}(72-6x-2z) \quad -(2)$$

$$z = \frac{1}{54}(110-x-y) \quad -(3)$$

1st iteration:

put $y=0$, $z=0$ in (1) to get

$$x_1 = \frac{1}{27}(85) = 3.1481$$

$$y_1 = \frac{1}{15}(72-6\times 3.1481) = 3.5408$$

$$z_1 = \frac{1}{54}(110-3.1481-3.5408) = 1.9132$$

2nd iteration:

$$x_2 = \frac{1}{27}(85-6\times 3.5408+1.9132) = 2.432$$

$$y_2 = \frac{1}{15}(72-6\times 2.432-2\times 1.9132) = 3.572$$

$$z_2 = \frac{1}{54}(110-2.432-3.572) = 1.926$$

3rd iteration:

$$x_3 = \frac{1}{27}(85-6\times 3.572+1.926) = 2.426$$

$$y_3 = \frac{1}{15}(72-6\times 2.426-2\times 1.926) = 3.573$$

$$z_3 = \frac{1}{54}(110-2.426-3.573) = 1.926$$

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4^{th} iteration:

$$x_4 = \frac{1}{27}(85 - 6 \times 3.573 + 1.926) = 2.425$$

$$y_4 = \frac{1}{15}(72 - 6 \times 2.425 - 2 \times 1.926) = 3.573$$

$$z_4 = \frac{1}{54}(110 - 2.425 - 3.573) = 1.926$$

$\therefore$ values of x, y, z are convergent in 3^{rd} & 4^{th} iterations.

$\Rightarrow$ Soln of given system is

$$x = 2.425,\quad y = 3.573,\quad z = 1.926$$

7. (a) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = 10$ and $y = 10$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 10) = x(10 - x)$ while the other three faces are kept at $0°$ C. Find the steady state temperature in the plate. [25]

Heat eqn is given by

$$\frac{\partial \phi}{\partial t} = k\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = 0$$

Using method of separation of variables,

$\phi = X \cdot Y \cdot T$ where X, Y, T are functions of x, y, t respectively

$$\Rightarrow \frac{1}{k} \cdot \frac{T'}{T} = \frac{X''}{X} + \frac{Y''}{Y} = -\lambda^2 \text{ (say)}$$

$$\Rightarrow T = e^{-k\lambda^2 t} \quad \text{--- (1)}$$

$$\frac{X''}{X} = -\lambda^2 - \frac{Y''}{Y} = -p^2 \text{ (say)}$$

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Never check it show

$$\Rightarrow X = c_1 \cos px + c_2 \sin px \quad -(2)$$

$$\frac{Y''}{Y} = -\lambda^2 - \frac{X''}{X} = -q^2 \text{ (say)}$$

$$\Rightarrow Y = c_3 \cos qy + c_4 \sin qy \quad -(3)$$

$$\& \quad p^2 + q^2 = \lambda^2 \quad -(4)$$

$$\Rightarrow \phi = (c_1 \cos px + c_2 \sin px)\cdot(c_3 \cos qy + c_4 \sin qy)\cdot e^{-k\lambda^2 t}$$

$$\phi(0, y) = 0 \Rightarrow c_1 = 0$$

$$\phi(x, 0) = 0 \Rightarrow c_3 = 0$$

$$\Rightarrow \phi = A \sin px \sin qy \, e^{-k\lambda^2 t}$$

$$\phi(10, y) = 0 \Rightarrow 10p = n\pi \Rightarrow p = \frac{n\pi}{10}$$

$$\cancel{\frac{\partial\phi}{\partial x} = Ap\cos px \sin qy \, e^{-k\lambda^2 t}}$$

$$\cancel{\frac{\partial\phi}{\partial y} = Aq \sin px \cos qy \, e^{-k\lambda^2 t}}$$

$$\cancel{\frac{\partial\phi}{\partial x} = 0 \text{ at } y = 0 \ \& \ y = 10 \Rightarrow 10q = m\pi \Rightarrow q = \frac{m\pi}{10}}$$

$$\cancel{\frac{\partial\phi}{\partial y} = 0 \text{ at } x = 0 \ \& \ x = 10 \text{ which is true as } p = \frac{n\pi}{10}}$$

PTO

$$\cancel{\Rightarrow \phi = \sum_{n=0}^{\infty} \Rightarrow \phi = A \sin\frac{n\pi}{10}x \sin\frac{m\pi}{10}y\, e^{-k\lambda^2 t}}$$

$$\cancel{\text{where } \lambda^2 = \frac{(n^2+m^2)\pi^2}{100}}$$

$$\Rightarrow \phi = \sum_{n=0}^{\infty} A_n \sin\frac{n\pi}{10}x \sin qy\; e^{-k\lambda^2 t}$$

Given $u(x, 10) = x(10-x)$

$$\Rightarrow \frac{10}{2} \cdot A_n = \int_0^{10} x(10-x) \cdot \sin\frac{n\pi}{10}x \, dx$$

$$\Rightarrow A_n = \frac{1}{5}\left[10 \int_0^{10} x \sin\frac{n\pi}{10}x\,dx - \left\{x^2 \times \frac{-10}{n\pi}\cos\frac{n\pi}{10}x\right|_0^{10} + \int_0^{10} 2x \times \frac{10}{n\pi}\cos\frac{n\pi}{10}x\right]$$

$$= \frac{1}{5}\left[10\left\{x \times \frac{-10}{n\pi}\cos\frac{n\pi x}{10}\Big|_0^{10} + \int_0^{10}\frac{10}{n\pi}\cos\frac{n\pi}{10}\,dx\right\} + 100 \times \frac{10}{n\pi} \times (-1)^n - \frac{20}{n\pi} \times \frac{10}{n\pi}\int_0^{10} \sin\frac{n\pi}{10}x\,dx\right]$$

$$= \frac{1}{5}\left[10\left(-\frac{100}{n\pi} \times (-1)^n + 0\right) + \frac{1000}{n\pi} \times (-1)^n + \frac{200}{n^2\pi^2} \times \frac{10}{n\pi}\cos\frac{n\pi}{10}x\Big|_0^{10}\right]$$

$$= \frac{400}{n^3\pi^3}\left[(-1)^n - 1\right]$$

$$\Rightarrow \phi = \sum_{n=0}^{\infty} \frac{400}{n^3\pi^3}\left[(-1)^n - 1\right] \sin\frac{n\pi}{10}x \sin qy\; e^{-k\lambda^2 t}$$

7. (b) Using modified Euler's method, find an approximate value of y when $x = 0.3$, given that $dy/dx = x + y$ and $y = 1$ when $x = 0$. (10)

$$f(x, y) = \frac{dy}{dx} = x + y$$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$\cancel{y(0.3) = y(0) + 0.3 \times f(0, 0)}$$

$$\cancel{= 1 + 0.3 \times}$$

$$y(0.1) = y(0) + 0.1 \times f(0, 1)$$

$$= 1 + 0.1 \times 1 = 1.1$$

$$y(0.2) = y(0.1) + 0.1 \times f(0.1, 1.1)$$

$$= 1.1 + 0.1 \times 1.2$$

$$= 1.1 + 0.12 = 1.22$$

$$y(0.3) = y(0.2) + 0.1 \times f(0.2, 1.22)$$

$$= 1.22 + 0.1 \times 1.42$$

$$= 1.22 + 0.142$$

$$= 1.362$$

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7. (c) For the given set of data points

$(x_1, f(x_1)), (x_2, f(x_2)) \ldots\ldots\ldots (x_n, f(x_n))$

write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula.

(15)

Step 1 : Start

Step 2 : Input $(x_1, f(x_1)), (x_2, f(x_2)) \ldots (x_n, f(x_n))$

Step 3 : Input value of x

Step 4 : Calculate

$$f(x) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \cdot f(x_1) + \frac{(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} f(x_2) + \cdots + \frac{(x-x_0)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} f(x_n)$$

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step 5: Print value of f(x)

step 6: stop.

8. (a) Determine the motion, of a spherical pendulum, by using Hamilton's equations. (16)