

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



Hyd-Ty

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TEST SERIES (MAIN)-2014

Test Code: LEVEL-I(M) IAS / T-04

MATHEMATICS

by K. VENKANNA

The person with 14 years of Teaching Experience

PDE, NA & CP, MECHANICS & FLUID DYNAMICS

126/250

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

- This question paper-cum-answer booklet has 50 pages and has 31 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- Symbols/notations carry their usual meanings, unless otherwise indicated.
- All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Varun Guntupalli

Roll No.

Test Centre

Hyderabad

Medium

English

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them.

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the Invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

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THIS SPACE**

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INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			- 09 -
	(b)			
	(c)			- 05 -
	(d)			
	(e)			
2	(a)			- 05 -
	(b)			- 10 -
	(c)			- 16 -
	(d)			- 06 -
3	(a)			- 05 -
	(b)			- 05 -
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4	(a)			
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5	(a)			- 09 -
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7	(a)			- 06 -
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	(c)			- 13 -
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				126 / 250

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Branch Office: 25A, Old Raisina Nagar Market, Delhi-110003
Ph. 611-48629967, 9999532811, 9999513762 | www.imsinstitute.com | www.terranetsschool.com | ims@ims21@gmail.com

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SECTION-A

- I. (a) Find the equation of the surface satisfying, $4yzp+q+2y=0$ and passing through $y^2+z^2=1$ and $x+z=2$.

$$\text{Given } 4yzp+q = -2y \quad \text{(1)}$$

\Rightarrow Lagrange's equations of (1) are

$$\frac{dx}{4yz} = \frac{dy}{1} = \frac{dz}{-2y} \quad \text{(2)}$$

$$\text{Considering } \frac{dx}{4yz} = \frac{dz}{-2y}$$

$$-dx = 2zdz$$

$$\Rightarrow x + z^2 = c_1 \quad \text{(3)}$$

$$\text{Considering } \frac{dy}{1} = \frac{dz}{-2y}$$

$$-2ydy = dz$$

$$\Rightarrow z + y^2 = c_2 \quad \text{(4)}$$

$$\Rightarrow \text{Solution of (1)} \quad z + y^2 = f(x + z^2)$$

& this passes through $y^2 + z^2 = 1$ & $x + z = 2$

$$\Rightarrow f(x + z^2) = (1 - z^2) + (2 - x) \\ = 3 - (x + z^2)$$

$$\Rightarrow f(x) = 3 - x$$

\Rightarrow Solution of (1) satisfying given conditions is

$$\text{Q9 } z + y^2 = 3 - (x + z^2)$$

$$\text{i.e. } \underbrace{x + z + y^2 + z^2 = 3}_{\text{---}}$$

(b) Solve $(x+y)(p+q)^n + (x-y)(p-q)^n = 1$

(19)

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Head Office: 189-190, Top Floor, Mukherjee Tower, Dr. Mukherjee Nagar, Delhi-110029.
Branch Office: 286, Old Rajender Nagar Market, Gurgaon-122009.
Ph: 011-46647787, 9999932511, 09999117626 | www.ims4maths.com | www.ims4mathskilling.com | Email: imsmaths2016@gmail.com

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(c) Using Newton Raphson Method, find all positive roots of the equation $10 \int_0^x e^{-t^2} dt - 1 = 0$ with six correct decimals.

$$\text{Let } f(x) = 10 \int_0^x e^{-t^2} dt - 1 \quad \text{--- (1)}$$

$$\Rightarrow f'(x) = 10e^{-x^2} \quad \text{--- (2)}$$

$$\text{Put } x_0 = 0$$

By ~~Newton~~ Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(-1)}{10} = 0.1$$

$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.1 - \frac{f(0.1)}{f'(0.1)} = 0.100336$$

$$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.1003357$$

$$\Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.1003357$$

~~∴ Root of $f(x) = 0$ with six correct
decimals, is $\underline{\underline{x = 0.100336}}$.~~

- (d) Design a circuit using NAND gates only that has one control line and three data lines. When control line is High the circuit should detect when one of the data lines has 1 on it. No more than one data line can have 1 on it. When control is Low the circuit output should be Low irrespective of input on data line. (10)

(e) Write the Hamiltonian function and equations of motion a compound pendulum.

(10)

2. (a) Solve $x^2r - y^2t + px - qy = \log x$ - ①

(10)

$$P+q = e^u, \quad r = e^v \quad - ②$$

$$\Rightarrow (D^2 - D'^2 + D - D') z = u \quad \text{where} \quad D = \frac{\partial}{\partial u}, \quad D' = \frac{\partial}{\partial v}$$

$$\Rightarrow (D - D')(D + D') z = u \quad - ③$$

$$\Rightarrow CF = \phi_1(v+x) + e^{-x} \phi_2(v)$$

$$\Rightarrow CF = \phi_1(v+u) + e^{-u} \phi_2(v-u) \quad - ④$$

$$\begin{aligned} PTE &= \frac{1}{(D - D')(D + D' + 1)} u \\ \text{Please check it} &= \frac{1}{(D + D' + 1)} \cdot \frac{1}{D} \cdot \left(1 + \frac{D'}{D}\right) u \\ &= \frac{1}{D + D' + 1} \cdot \frac{1}{D} \cdot u \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{D+D'+1} \cdot \frac{u^2}{2} \\
 &= [1 - (D+D') + (D+D)^2] \frac{u^2}{2} \\
 &= \frac{u^2}{2} - u + 1
 \end{aligned}$$

\Rightarrow Solution of ① is

$$Z = \phi_1(v+u) + e^{-u}\phi_2(v-u) + \frac{u^2}{2} - u + 1$$

~~$$\text{i.e. } Z = \phi_1(xy) + \phi_2(\frac{y}{x}) + \frac{(\log x)^2}{2} \log x + 1$$~~

where ϕ_1, ϕ_2 are arbitrary functions.

- (b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it. (15)

Given equation is $s + s + t = 0$ - ①

$\rightarrow \lambda$ -quadratic of ① is $\lambda^2 + \lambda + 1 = 0$

$$\rightarrow \lambda = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\frac{dy}{dx} + \lambda = 0 \quad \text{gives}$$

$$y + \left(\frac{-1 + \sqrt{3}i}{2}\right)x = c_1, \quad y + \left(\frac{-1 - \sqrt{3}i}{2}\right)x = c_2$$

$$\text{Put } \xi = y - \frac{x}{2} \quad \& \quad \eta = \frac{\sqrt{3}x}{2}$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = -\frac{1}{2} \frac{\partial z}{\partial \xi} + \frac{\sqrt{3}}{2} \frac{\partial z}{\partial \eta}$$

$$\begin{aligned}
 \frac{\partial^2 z}{\partial x^2} &= \left(-\frac{1}{2} \frac{\partial^2 z}{\partial \xi^2} + \frac{\sqrt{3}}{2} \frac{\partial^2 z}{\partial \xi \partial \eta}\right) \cdot -\frac{1}{2} + \left(-\frac{1}{2} \frac{\partial^2 z}{\partial \eta^2} + \frac{\sqrt{3}}{2} \frac{\partial^2 z}{\partial \xi \partial \eta}\right) \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{1}{4} \frac{\partial^2 z}{\partial \xi^2} - \frac{\sqrt{3}}{2} \frac{\partial^2 z}{\partial \xi \partial \eta} + \frac{3}{4} \frac{\partial^2 z}{\partial \eta^2} - ②
 \end{aligned}$$

$$\begin{aligned}\frac{\partial^2 z}{\partial x \partial y} &= \widehat{\frac{\partial z}{\partial y}}(\widehat{\frac{\partial x}{\partial y}}) = \widehat{\frac{\partial z}{\partial y}} \cdot \widehat{\frac{\partial x}{\partial y}} \\ &= \left(-\frac{1}{2} \frac{\partial^2 z}{\partial \xi^2} + \frac{\sqrt{3}}{2} \frac{\partial^2 z}{\partial \eta \partial \xi} \right) \cdot 1 + 0 \\ &= -\frac{1}{2} \frac{\partial^2 z}{\partial \xi^2} + \frac{\sqrt{3}}{2} \frac{\partial^2 z}{\partial \xi \partial \eta} \quad - (3)\end{aligned}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$$

$$= \frac{\partial z}{\partial \xi}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial \xi^2} \quad - (4)$$

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 Branch Office: 25A, Chh Ramlal Nagar Market, Delhi-110069
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(c) Solve the equations:

(18)

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 - 10x_4 = -9$$

Using Gauss-Siedel iterative method,

$$x_1 = \frac{1}{10} (3 + 2x_2 + x_3 + x_4) \quad \text{--- (1)}$$

$$x_2 = \frac{1}{10} (15 + 2x_1 + x_3 + x_4) \quad \text{--- (2)}$$

$$x_3 = \frac{1}{10} (27 + x_1 + x_2 + 2x_4) \quad \text{--- (3)}$$

$$x_4 = \frac{1}{10} (9 - x_1 - x_2 - 2x_3) \quad \text{--- (4)}$$

Put $x_2 = x_3 = x_4 = 0$ in (1)

$$x_2 = \frac{1}{10} (15 + 2 \times 0.3) = 1.56$$

$$x_3 = \frac{1}{10} (27 + 0.3 + 1.56) = 2.886$$

$$x_4 = \frac{1}{10} (9 - 0.3 - 1.56 - 2 \times 2.886) = 0.1368$$

2nd iteration:

$$x_1 = \frac{1}{10} (3 + 2 \times 1.56 + 2.886 + 0.1368) = 0.91428$$

$$x_2 = \frac{1}{10} (15 + 2 \times 0.91428 + 2.886 + 0.1368) = 1.985136$$

$$x_3 = \frac{1}{10} (27 + 0.91428 + 1.985136 + 2 \times 0.1368) = 3.0173$$

$$x_4 = \frac{1}{10} (9 - 0.91428 - 1.985136 - 2 \times 0.1368) = 0.0066$$

3rd iteration:

$$x_1 = \frac{1}{10} (3 + 2 \times 1.985136 + 3.0173 + 0.0066) = 0.9994$$

$$x_2 = \frac{1}{10} (15 + 2 \times 0.9994 + 3.0173 + 0.0066) = 2.0023$$

$$x_3 = \frac{1}{10} (27 + 0.9994 + 2.0023 + 2 \times 0.0066) = 3.0015$$

$$x_4 = \frac{1}{10} (9 - 0.9994 - 2.0023 - 3.0015) = 0$$

4th iteration:

$$x_1 = \frac{1}{10} (3 + 2 \times 2.0023 + 3.0015 + 0) = 1.0006$$

$$x_2 = \frac{1}{10} (15 + 2 \times 1.0006 + 3.0015) = 2.0003$$

$$x_3 = \frac{1}{10} (27 + 1.0006 + 2.0003) = 3.0001$$

$$x_4 = \frac{1}{10} (9 - 1.0006 - 2.0003 - 3.0001) = 0$$

\Rightarrow Solution of the equations is

$$x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 0$$

(d) Convert the following:

(07)

(i) $(41.6875)_{10}$ to binary number(ii) $(101101)_2$ to decimal number(iii) $(AF63)_{16}$ to decimal number

$$\begin{aligned}
 \text{(i)} \quad & 2 \overline{) 41} \quad \Rightarrow (41)_{10} = (101001)_2 \\
 & 2 \overline{) 20} - 1 \\
 & 2 \overline{) 10} - 0 \\
 & 2 \overline{) 5} - 0 \\
 & 2 \overline{) 2} - 1 \\
 & 1 - 0 \\
 & 0.6875 = 0.5 + 0.1875 \\
 & = 0.5 + 0 \times 0.25 + 1 \times 0.125 + 0.0625 \\
 & = 1 \times \frac{1}{2} + 0 \times \frac{1}{2^2} + 1 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4}
 \end{aligned}$$

$$\Rightarrow (41.6875)_{10} = (101001.1011)_2$$

$$\begin{aligned}
 \text{(ii)} \quad (101101)_2 &= 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 \\
 &= 32 + 8 + 4 + 1 \\
 &= 45
 \end{aligned}$$

06

✓

$$\begin{aligned}
 (\text{iii}) \quad & (170)^2 / 16 = 170 \cdot 170 / 16 = 170 \cdot 10.625 \\
 & = 40960 + 3840 + 96 + 3 \\
 & = 44899.
 \end{aligned}$$

3. (a) Form a partial differential equation by eliminating the function f from $x = y^2 + 2f(1/x + \log y)$. — ① (07)

$$\Rightarrow P = \frac{\partial z}{\partial x} = 2f' \left(\frac{1}{x} + \log y \right) \cdot \frac{1}{x^2} - ②$$

$$q = \frac{\partial z}{\partial y} = 2y + 2f' \left(\frac{1}{x} + \log y \right) \cdot \frac{1}{y} - ③$$

\Rightarrow From ② & ③

$$q = 2y + (-P x^2) \cdot \frac{1}{y}$$

$$P x^2 + q y = -2y^2$$

is the required
differential equation.

- (b) Find the function $f(x, y)$ which satisfies the Laplace's equation inside the rectangle $0 \leq x \leq \pi, 0 \leq y \leq a$ subject to the boundary conditions.

$$f(0, y) = f(\pi, y) = 0, 0 < y < a$$

$$f(x, a) = a \text{ for } 0 \leq x \leq \pi \text{ and } f(x, 0) = x(\pi - x) \text{ for } 0 \leq x \leq \pi. \quad (18)$$

Laplace's equation is given by

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \quad \text{--- (1)}$$

Using separation of variables, let

$$f(x, y) = X(x) \cdot Y(y)$$

$$\Rightarrow \text{From (1), } \frac{X''}{X} = -\frac{Y''}{Y} = -\lambda^2 \quad (\text{say})$$

$$\Rightarrow X(x) = A \cos \lambda x + B \sin \lambda x$$

$$Y(y) = C \cosh \lambda y + D \sinh \lambda y$$

$$\Rightarrow f(x, y) = (A \cos \lambda x + B \sin \lambda x) \cdot (C \cosh \lambda y + D \sinh \lambda y) \quad \text{--- (2)}$$

Given $f(x,y) = B \sin nx \cdot (C \cosh ny + D \sinh ny)$

\Rightarrow From ②, $A = 0$

$$\lambda \pi = n\pi \quad \text{where } n \in \mathbb{N}$$

$$\Rightarrow \lambda = n$$

$$\Rightarrow f(x,y) = B \sin nx \cdot (C \cosh ny + D \sinh ny) - ③$$

Also given $f(x,a) = a$ for $0 \leq x \leq \pi$

$$\therefore f(x,a) = x(\pi-x) \quad \text{for } 0 \leq x \leq \pi$$



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Branch Office: 269, Old Raja Bazar, New Market, Delhi-110003

Ph. 011-45423047, 0989929111, 09999197625 | www.imsimathematics.com | www.imsimathematics.org | Email: imsimaths@gmail.com

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(c) Given the values

x	: 5	7	11	13	17
$f(x)$: 150	392	1452	2366	5202

evaluate $f(9)$, using

(i) Lagrange's formula

(ii) Newton's divided difference formula.

(i) Lagrange's formula

$$f(x) = \sum \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} f(x_0)$$
$$\Rightarrow f(9) = \frac{2 \times (-2) \times (-4) \times (-8)}{(-2) \times (-6) \times (-8) \times (-12)} \times 150 + \frac{4 \times (-2) \times (-4) \times (-8)}{2 \times (-4) \times (-6) \times (-10)} \times 392 + \frac{4 \times (2) \times (-4) \times (-8)}{6 \times 4 \times (-2) \times (-6)} \times 1452$$



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(15)

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$$\begin{aligned}
 & + \frac{4 \cdot 2 \cdot (-2) \cdot (-8)}{8 \cdot 6 \cdot 2 \cdot (-4)} \times 2366 \\
 & + \frac{4 \cdot 2 \cdot (-2) \cdot (-4)}{12 \cdot 10 \cdot 6 \cdot 4} \times 5202 \\
 = & -16 \cdot 6667 + 209 \cdot 0667 + 1290 \cdot 6667 \\
 & - 788 \cdot 6667 + 115 \cdot 6 \\
 = & \underline{810}
 \end{aligned}$$

(ii)

x	f(x)		
5	150		
7	792	$242/2 = 121$	24
11	1452	$1060/4 = 265$	26
13	2366	$914/2 = 457$	22
17	5202	$2836/4 = 709$	42

~~$\Rightarrow f(9) =$~~

$$\begin{aligned}
 f(x) = & f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0) \cdot (x-x_1) \\
 & \cdot f(x_0, x_1, x_2) \\
 & + \dots
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow f(9) = & 150 + (9-5) \times 121 + (9-7) \times 24 \\
 & + (9-5) \times (9-7) \times (9-11) \times 1
 \end{aligned}$$

~~14~~ 810.

$$AB + \bar{C}(\bar{A} + \bar{D}) = AB + BD + \bar{BD} + \bar{ACD}$$

$$\text{Consider } BD + \bar{BD} + \bar{ACD} - \bar{C}(\bar{A} + \bar{D})$$

$$= BD + \bar{BD} + \bar{ACD} - \bar{AC} - \bar{CD} \quad [\because \bar{CA} = \bar{AC}]$$

$$= BD + \bar{BD} + \cancel{\bar{AC}(D-1)} - \cancel{\bar{CD}}$$

$$= BD + \bar{BD} + \bar{ACD} - \bar{CD}$$

$$= 1 + \bar{ACD} - \bar{CD}$$

Please
try it again
check pt
from key
No marks

P.T.O.



Head Office: 125-126, Top Floor, Noida Expressway, Dr. Mahatma Gandhi Marg, Delhi-110089.
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SECTION-B

5. (a) Solve $p \cos(x+y) + q \sin(x+y) = z$ - ① (10)

Lagrange's equations of ① are

$$\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z} - ②$$

$$\Rightarrow \frac{d(x+y)}{\sin(x+y) + \cos(x+y)} = \frac{dz}{z}$$

$$\text{Put } x+y = v$$

$$\Rightarrow \frac{dv}{\sin v + \cos v} = \frac{dz}{z}$$

$$\text{put } \tan \frac{v}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{v}{2} dv = dt$$

$$\Rightarrow dv = \frac{2dt}{1+t^2}$$

$$\Rightarrow \frac{2dt/(1+t^2)}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} = \frac{dz}{z} \Rightarrow \frac{2dt}{2-(t-1)^2} = \frac{dz}{z}$$

$$\Rightarrow \ln z = \sqrt{2} \tanh^{-1} \left(\frac{x+y}{\sqrt{2}} \right) + C_1$$

$$\Rightarrow C_1 = \ln z - \sqrt{2} \tanh^{-1} \left(\frac{\tan \left(\frac{x+y}{2} \right) - 1}{\sqrt{2}} \right) - ③$$

$$\text{From } ③, \quad \frac{dy}{dx} = \tan(x+y) \Rightarrow \frac{d(x+y)}{dx} = 1 + \tan(x+y)$$

$$\text{Let } x+y = w$$

$$\Rightarrow \frac{dw}{dx} = 1 + \tan w \Rightarrow dx = \frac{dw}{1 + \tan w}$$

$$\Rightarrow dx = \frac{\cos w dw}{\sin w + \cos w} = \frac{\cos w (\cos w - \sin w)}{\cos 2w} dw$$

$$= \frac{\frac{1}{2}(\cos 2w + 1) - \frac{1}{2}\sin 2w}{\cos 2w} dw$$

$$\Rightarrow x = \frac{1}{2} \left[w + \frac{1}{2} \ln(\sec 2w + \tan 2w) - \frac{1}{2} \ln \sec 2w \right] + C_2$$

$$= \frac{1}{4} [2w + \ln(1 + \sin 2w)] + C_2 - ④$$

\Rightarrow Solution of ① is

$$\Phi \left[x - \frac{1}{4} [2(x+y) + \ln(1 + \sin 2(x+y))] \right], \quad \ln z - \sqrt{2} \tanh^{-1} \left(\frac{\tan \left(\frac{x+y}{2} \right) - 1}{\sqrt{2}} \right) = 0.$$

(b) Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. - ①

(10)

Auxiliary equation of ① is

$$m^2 - m - 2 = 0$$

$$\text{i.e. } (m-2)(m+1) = 0$$

$$\Rightarrow m = 2, -1$$

$$\Rightarrow CF = \phi_1(y-x) + \phi_2(y+2x) - ②$$

$$P.I. = \frac{1}{(D-2D')(D+D')} (y-1)e^x$$

$$= e^x \cdot \frac{1}{(D+1-2D')(D+1+D')} (y-1)$$

$$= e^x \cdot \frac{1}{(D+1-2D')} (1 - (D+D')) (y-1)$$

$$= e^x \cdot \frac{1}{1 + D - 2D^1} (y-1-1)$$

$$= e^x \cdot (1 - (D - 2D^1)) (y-2)$$

$$= e^x (y-2+2)$$

$$= ye^x$$

\Rightarrow Solution of ① is

$$Z = \phi_1(y-x) + \phi_2(y+2x) + ye^x$$

where ϕ_1, ϕ_2 are arbitrary functions of x, y

From the following data:

Wages (Rs.)	Below 40	40-60	60-80	80-100	100-120
No. of Persons (in thousands)	250	120	100	70	50

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Branch Office: 256, C-Block, Rajender Nagar Market, Delhi-110049
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PTO

- (d) A rod, of length $2a$, revolves with uniform angular velocity ω to about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α , show that $\omega^2 = 3g / (4a \cos \alpha)$.

Prove also that direction of reaction at the hinge makes with the vertical an angle $\tan^{-1} \left(\frac{3}{4} \tan \alpha \right)$. (10)

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$$u = x/(1+t), v = y/(1+t), w = z/(1+t).$$

(10)

Streamlines are given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\text{i.e. } \frac{dx}{\frac{x}{1+t}} = \frac{dy}{\frac{y}{1+t}} = \frac{dz}{\frac{z}{1+t}}$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y} \quad \& \quad \frac{dy}{y} = \frac{dz}{z}$$

$$\Rightarrow \frac{y}{x} = c_1 \quad \& \quad \frac{z}{y} = c_2$$

\Rightarrow Streamlines are given by

$$t\left(\frac{y}{x}, \frac{z}{y}\right) = 0 \quad \text{where } t \text{ is an arbitrary function}$$

Pathlines are given by

$$\frac{dx}{dt} = \frac{x}{1+t}, \quad \frac{dy}{dt} = \frac{y}{1+t}, \quad \frac{dz}{dt} = \frac{z}{1+t}$$

$$\Rightarrow \ln x = \ln(1+t) + d_1, \quad \ln y = \ln(1+t) + d_2, \quad \ln z = \ln(1+t) + d_3$$

Pathlines are given by

$$\frac{x}{1+t} = d_1, \quad \frac{y}{1+t} = d_2, \quad \frac{z}{1+t} = d_3$$

where d_1, d_2, d_3 are arbitrary constants.

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Branch Office: 258, Old Rajender Nagar Market, RoD-533066

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7. (a) Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$, compute $y(0.2)$, $y(0.4)$ and $y(0.6)$ by Runge-Kutta method of fourth order.

(17)

Runge-Kutta method of 4th order is given by

$$y_{n+1} = y_n + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$$\text{where } k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = f(x_n + h, y_n + k_3)$$

$$\text{Let } f(x, y) = \frac{dy}{dx} = x^3 + y$$

$$y(0) = 2 ; h = 0.2$$

$$\Rightarrow k_1 = f(0, 2) = 0 + 2 = 2$$

$$k_2 = f(0 + 0.1, 2 + \frac{2}{2}) = f(0.1, 3) = 3.001$$

$$k_3 = f(0 + 0.1, 2 + \frac{3.001}{2}) = f(0.1, 3.5005) \\ = 3.5015$$

$$k_4 = f(0.2, 2 + 3.5005) = 5.5095$$

*Please
check the
calculations*

$$y_{0.2} = y_0 + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \\ = 5.4191$$

$$y_{0.4} = y_{0.2} + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \\ \Rightarrow k_1 = f(x_{0.2}, y_{0.2}) = f(0.2, 5.4191) = 5.4271 \\ k_2 = f(0.3, y_{0.2} + \frac{5.4271}{2}) = f(0.3, 8.15965) = 8.15965 \\ k_3 = f(0.3, y_{0.2} + \frac{8.15965}{2}) = f(0.3, 9.525925) = 9.525925 \\ k_4 = f(0.4, y_{0.2} + 9.525925) = f(0.4, 15.09025) = 15.09025 \\ \Rightarrow y_{0.4} = 5.4191 + 9.3012 = 14.7203$$

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$$y_{0.6} = y_{0.4} + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$$k_1 = b(x_{0.4}, y_{0.4}) = b(0.4, 14.7203) = 14.7843$$

$$k_2 = b(0.5, y_{0.4} + \frac{14.7843}{2}) = b(0.5, 22.11245) = 22.23745$$

$$k_3 = b(0.5, y_{0.4} + \frac{22.23745}{2}) = b(0.5, 25.83925) = 25.964025$$

$$k_4 = b(0.6, y_{0.4} + 25.964025) = b(0.6, 40.684725) = 40.900325$$

$$\Rightarrow y_{0.6} = 14.7203 + 25.34793$$

$$= \underline{\underline{40.0683}}$$

$$\Rightarrow y(0.2) = 5.4191$$

$$y(0.4) = 14.7203$$

$$y(0.6) = 40.0683$$

- (b) A The velocity v of a particle at distance s from a point on its path is given by the table:

(15)

s ft	0	10	20	30	40	50	60
v ft/sec	47	58	64	65	61	52	38

Estimate the time taken to travel 60ft by using simpson's $\frac{1}{3}$ rule. Compare the result with Simpson's $\frac{3}{8}$ rule.

s ft : 0 10 20 30 40 50 60

$\frac{1}{v}$ sec : $\frac{1}{47}$ $\frac{1}{58}$ $\frac{1}{64}$ $\frac{1}{65}$ $\frac{1}{61}$ $\frac{1}{52}$ $\frac{1}{38}$

~~approximate~~ $y_0, y_1, y_2, y_3, y_4, y_5, y_6$

Simpson's $\frac{1}{3}$ rule :

$$t = \frac{h}{3} (y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4))$$

$$= \frac{10}{3} \left[\frac{1}{47} + \frac{1}{38} + 4 \left(\frac{1}{58} + \frac{1}{64} + \frac{1}{65} \right) + 2 \left(\frac{1}{61} + \frac{1}{52} \right) \right]$$

$$= 1.0635211 \text{ sec.}$$

Simpson's $\frac{3}{8}$ rule:

$$t = \frac{3h}{8} [y_0 + y_6 + 3(y_1 + y_4) + 3(y_2 + y_5) + 2y_3]$$

$$= \frac{3 \times 10}{8} \left[\frac{1}{47} + \frac{1}{38} + 3 \left(\frac{1}{58} + \frac{1}{61} \right) + 3 \left(\frac{1}{60} + \frac{1}{52} \right) + 2 \times \frac{1}{65} \right]$$

$$= 1.064375$$

→ Time taken to travel 60ft estimate by

Simpson's $\frac{1}{3}$ rule is 1.0635211 sec

& by Simpson's $\frac{3}{8}$ rule is 1.064375 sec .



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P.T.O.

(c) Design an algorithm and draw a flow chart for Regula falsi method.

(18)

Regula falsi method is given by

$$x_{n+2} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$$

Algorithm:

Step 1: Input $f(x)$, x_0 , x_1 , error (ϵ), no. of iterations (n)

Step 2: Calculate $f(x_0)$, $f(x_1)$ & put $i=0$

Step 3: If $|f(x_0)| < \epsilon$, print x_0 is the root & end

Step 4: If $|f(x_1)| < \epsilon$, print x_1 is the root & end

Step 5: If $f(x_0) \cdot f(x_1) > 0$, print "invalid entries of x_0, x_1 " & end

$$\text{Step 6: Put } x_2 = \frac{x_0 \cdot f(x_1) - x_1 \cdot f(x_0)}{f(x_1) - f(x_0)}$$

Step 7: If $f(x_0) \cdot f(x_2) < 0$ put
 Step 7: If $|f(x_2)| < \epsilon$, print x_2 is a root & end
 Step 8: If $f(x_0) \cdot f(x_2) < 0$ put $x_1 = x_2$ else
 put $x_0 = x_2$ goto step 6
 Step 9: $i = i + 1$
~~Step 9: Stop~~
 Step 10: If $i < n$ goto step 6 else print "Not
 Given x_0, x_1 , the solution is not converging"
 & end
 Step 11: Stop.

Flowchart

