

159/250

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



(INSTITUTE OF MATHEMATICAL SCIENCES)

TEST SERIES (MAIN)-2014

Test Code: 8-AUG-14-MAS(M) Test - 2

**MATHEMATICS**

by K. VENKANNA

The present syllabus of Teaching Examinations

PAPER-II: Algebra, Complex Analysis, Real Analysis & LPP

Time: Three Hours

Maximum Marks: 250

**INSTRUCTIONS**

This question paper consists of two parts containing 56 pages and has

42 MARKS/SUPPLEMENTARY MARKS. The maximum length of the question paper is 100 mm. The answer booklet you have received contains all the questions.

Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate spaces now, but on the right side.

A consolidated Question Paper-cum-Answer Booklet, having space below each part/one mark or a question shall be provided to them for writing the answers. Candidates shall be required to enter the answers to the parts/one mark of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the admission Certificate.

Candidates should attempt Questions Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.

9. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notebooks carry their usual meanings, unless otherwise indicated.

10. All questions carry one mark.

11. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.

12. All rough work should be done in the space provided and covered out finally.

13. The candidate should respect the instructions given by the Invigilator.

14. The question paper-cum-answer booklet must be returned only to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.

**READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**

Name:

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Roll No.:

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I have read all the instructions and shall abide by them.

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**IMPORTANT NOTE:**

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P.T.O.

**SECTION - A**

Q. (a) Give an example of an infinite group that has exactly two elements of order 1.

(10)

**Q. NO.**                      **Marks**

1(b) → 06

1(c) → 09

1(d) → 08

1(e) → 09

2(a) → 08

2(b) → 06

2(c) → 13

2(d) → 16

3(c) → 18

5(a) → 08

5(c) → 19

5(d) → 09

5(e) → 03

8(b) → 14

8(c) → 09

8(d) → 14

Total → 159 | 250

P.T.O.

- (d) If  $u = v - (x-y)$  ( $x \neq y$ ,  $y \neq 0$ ) and  $f(z) = u + vi$  is an analytic function of  $z = x+iy$ , find  $f(z)$  in terms of  $x$  and  $y$ .

$$\text{Def} \quad G(z) = (u+i)(u+iv) = (u-v) + i(u+v) = u + iv. \quad (10)$$

$$J = (x-y)(x^2+xy+y^2)$$

$$\text{we have } g'(z) = U_x + iV_x = U_x - iU_y.$$

From Milne-Thompson method

$$G(2) = D_*(2,0) - i(U_Y(2,0))$$

$$U_1 = -y(x^2 + 4xy + y^2) + (x-y)(2x+4y)$$

$$U_n(2,0) = 3 \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = 2 \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$Uy = -(x^2 + 4xy + y^2) + (x - 1)(4x + 2y)$$

$$W_2(z_{1,0}) = -z^2 + 4z^2 = 3z^2$$

$$g^4(3) = -2z^2 - i \bar{z} \bar{z}^2 = (-2 - 3i)^{-2}$$

Integrating with respect to  $z$  we get

$$f(3) = (2 - 3i) \frac{3^3}{3} + c$$

$$f(3) = \frac{(-i)}{3} \left[ \left(\frac{2-3i}{2}\right) 3^3 + (-)\right]$$

$$= \left( \frac{-1 - 51}{6} \right) x^3 + c'$$

$$f(z) = (-1 - 5i) \frac{z^3}{6} + c$$

(e) Write the dual of the following LPP.

$$\text{Maximize } Z = 2x_1 + 5x_2 + 3x_3$$

Subject to

$$2x_1 + 2x_2 - 3x_3 \leq 8$$

$$-2x_1 - 2x_2 + 3x_3 \geq -7$$

$$x_1 + 3x_2 - 5x_3 \leq 2$$

$$4x_1 + x_2 + 3x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

(10)

Primal: Maximize  $Z = 2x_1 + 5x_2 + 3x_3$ 

$$2x_1 + 4x_2 - 3x_3 \leq 8$$

$$2x_1 + 2x_2 - 3x_3 \leq 7$$

$$-x_1 - 3x_2 + 5x_3 \leq 2$$

$$4x_1 + x_2 + 3x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

After converting to less than or equal to constraint

proceeding to write dual

Dual: Minimize  $Z = 8y_1 + 7y_2 + 2y_3 + 4y_4$ 

$$2y_1 + 2y_2 - y_3 + 4y_4 \geq 2$$

$$4y_1 + 2y_2 - 3y_3 + y_4 \geq 5$$

$$-3y_1 - 3y_2 + 5y_3 + 3y_4 \geq 3$$

where  $y_1, y_2, y_3, y_4 \geq 0$ .

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P.T.O.

2. (a) Suppose  $G$  is the group defined by the following Cayley table.

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	3	4	6	5	7	8
3	3	4	5	6	7	8	1	2
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	4	3	2	1	8	7
7	7	8	1	2	3	4	5	6
8	8	7	6	5	4	3	2	1

- (i) Find the centralizer of each member of  $G$ .  
(ii) Find  $Z(G)$ .  
(iii) Find the order of each element of  $G$ . How are these orders arithmetically related to the order of the group?

Given  $G = \{1, 2, 3, 4, 5, 6, 7, 8\}$ .

$$\begin{aligned} (i) \quad Z(G) &= \{x \in G \mid gx = xg \forall g \in G\} \\ &= \{1, 5\} \end{aligned}$$

(ii.) centralizer of each member of  $G$  are as follows:

$$\text{for } 1 = G \quad 5 = G$$

$$2 = \{1, 2, 5, 6\} \quad 6 = \{1, 2, 5, 6\}$$

$$3 = \{1, 2, 5, 7\} \quad 7 = \{1, 3, 5, 7\}$$

~~$$- 08 \quad 4 = \{1, 4, 5, 8\} \quad 8 = \{1, 4, 5, 8\}$$~~

$$\begin{array}{llll} (iii) \quad o(1) = 1 & 3^2 = 5 & o(5) = 1 & 7^2 = 5 \\ o(2) = 2 & 3^3 = 7 & o(6) = 2 & 7^3 = 3 \\ o(3) = 4 & 3^4 = 1 & o(7) = 4 & 7^4 = 1 \\ o(4) = 2 & & o(8) = 2 & \end{array}$$

The order of elements are factors of the group order which is 8.

- (b) Consider the element  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  in  $SL(2, \mathbb{R})$ . What is the order of  $A$ ? If we view  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  as a member of  $SL(2, \mathbb{Z}_p)$  ( $p$  is a prime), what is the order of  $A$ ?

$$\text{Where } SL(2, \mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in GL(2, \mathbb{R}) \mid ad - bc = 1 \right\} \quad (18)$$

Order of  $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  is infinity as

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \text{ and } n \text{ is not equal to zero.}$$

Let  $A \in SL(2, \mathbb{Z}_p)$

Now  $A^{p-1} = \begin{pmatrix} 1 & p-1 \\ 0 & 1 \end{pmatrix}$

But  $A^p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Order of  $A$  in  $SL(2, \mathbb{Z}_p)$  is  $p$

(c) Show that  $\int x^{n-1} e^{-x} dx$  converges iff  $n > 0$ .

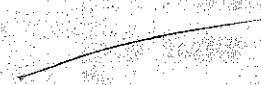
(16)

Consider the breakup of integral as

$$\int_0^a x^{n-1} e^{-x} dx + \int_a^\infty x^{n-1} e^{-x} dx \text{ where } a > 0$$

Convergence at  $x=0$ :For  $n > 1$  the integral is proper integral.Let  $n < 1$ ,

$$\text{let } f(x) = x^{n-1} e^{-x}$$

$$g(x) = x^{-1}$$


$$\lim_{x \rightarrow 0^+} \frac{f(x)}{g(x)} = 1 \text{ which is non-zero}$$

finite. Therefore by comparison test

$\int f(x)dx$  and  $\int g(x)dx$  both converge and diverge together.

$$\int \frac{1}{x^n} dx \text{ converges iff } -n < 1 \Rightarrow n > 0$$

$$\int x^{n+1} e^{-x} dx \text{ converges if } n > 0 \text{ and diverges for } n \leq 0$$

Convergence at  $x \rightarrow \infty$ :

$$\text{Consider } g(x) = \frac{1}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{x^{n+1} e^{-x}}{x^{-2}} = \lim_{x \rightarrow \infty} \frac{x^{n+1} e^{-x}}{\frac{1}{x^2}} = 0$$

∴ By comparison test we have  $\int x^{n+1} e^{-x} dx$  converges for all values irrespective of  $n$  since

$$\int_0^\infty \frac{1}{x^2} dx \text{ is convergent.}$$

thus  $\int_1^{\infty} e^{-x} n^{-1} dx$  is convergent iff  $n > 0$ .

- (ii) Show by the method of contour integration that

$$\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 0. \quad (17)$$

Consider the integral  $\int_C \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$

where  $C$  is the circle  $|z|=1$ , on which

$$\begin{aligned} z &= e^{i\theta} \\ dz &= i e^{i\theta} d\theta \\ \int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta &= \int_0^{2\pi} \frac{1+e^{i\theta}+e^{-i\theta}}{5+2e^{i\theta}+2e^{-i\theta}} d\theta \\ &= \int_{|z|=1} \frac{1+(z+\bar{z})}{5+2(z+\bar{z})} \frac{-i}{z} dz \end{aligned}$$

$$\underset{|z|=1}{\oint} \frac{e^z z^2 + z + 1}{2z^2 + 5z + 2} dz$$

By Cauchy-Riemann theorem, we have

$$\int f(z) dz = 2\pi i \cdot \text{Res}$$

where  $\text{Res}$  is the sum of residues of poles

lying inside the circle  $|z| < 1$ , e.g.  $|z|=1$ .

$\therefore$  Poles of  $(2z^2 + 5z + 2)z \cdot f(z)$  are given by

$$(2z^2 + 5z + 2)z = 0$$

$$\Rightarrow z=0, z = \frac{-5 \pm \sqrt{25 - 16}}{4} = \frac{-5 \pm 3}{4}$$

Out of these only  $z=0$  and  $z = -\frac{1}{2}$  lie inside

$$|z|=1$$

$$\text{Residue at } z=0 \text{ is } \lim_{z \rightarrow 0} \frac{-i(z^2 + z + 1)}{2z(z^2 + \frac{5}{2}z + 1)} = \frac{-i}{2}$$

$$\text{Residue at } z = -\frac{1}{2} \text{ is } \lim_{z \rightarrow -\frac{1}{2}} \frac{(z + \frac{1}{2})(-i)(z^2 + z + 1)}{2z(z + \frac{1}{2})(z + 2)} = \frac{i}{2}$$

$$\text{Sum of residues} = \frac{1}{2} - \frac{i}{2} = 0.$$

$$\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 2\pi \cdot \left(\frac{1}{2}\right) = \pi$$

$$\Rightarrow \int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = \pi$$

$$\Rightarrow \int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = \pi$$

(a) Give an example of a field of 9 elements

(15)

P.T.O.



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$$f_n(n) = \overbrace{n^n (1-x)}^{\text{cancel}}$$

~~front~~  $(1-x)^n$  Question printed wrong

(b) Show that the sequence of functions  $\{f_n\}_{n=1}^{\infty}$ , where  $f_n(x) = n \times I((1-x)^n)$ , is not uniformly convergent on  $[0, 1]$ .

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The  $\langle f_n(x) \rangle$  is pointwise convergent to

$$\lim_{x \rightarrow 0^+} f(x) = 0 \quad \text{in } (0, 1].$$

But when  $n=0$ , the sequence diverges to  $\infty$

For a sequence of function to be uniformly convergent we have the point function

continuous over the closed interval.

When the point-wise function is not continuous it can be concluded that  $f_{n_0}$  is

not uniformly convergent



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PTO

- (c) A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 tons respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 300 units respectively. Unit transportation costs (in rupees) are given below:

	D	E	F	G	H
A	5	8	6	6	3
From	B	4	7	7	5
C	8	4	6	5	4

Determine an optimum distribution for the company in order to minimize the total transportation cost.

$$\text{Supply of plants} = 800 + 500 + 900 = 2200$$

$$\begin{aligned} \text{Demand of warehouses} &= 400 + 400 + 500 + 400 + 300 \\ &= 2200 \end{aligned}$$

Given problem is unbalanced. To make it balanced we add a plant P of capacity 300 with zero cost.

To find basic feasible solution by Vogel's

Approximation method:

Differences

	D	E	F	G	H	
A	5	8	6	6	3	(800)
B	4	7	7	5	X	500
C	8	4	6	5	X	500
P	0	0	0	0	0	0
	X	X	(300)	X	X	300
	400	400	500	400	800	
	(4)	(4)	(6)	(6)	(6)	
	(1)	(3)	(0)	(0)	(1)	
	(1)	(0)	(0)	(0)	(1)	
	(4)	(1)	(0)			

PTO

The allocations are  $T$  which is one less than  $m+n-1 = 5+4-1 = 8$ . So assigning zero value to cell  $(4,5)$  which forms a ~~an~~ open loop.

Basic feasible solution is obtained. Proceeding for  $uv$  method to find optimal basic feasible solution.

					$y_j$
5	8	6	6	2	$u_1 = -3$
-4	-7	-3	-3	800	
4	7	7	6	5	$v_2 = 0$
1000	-3	-1	100	1	
8	4	6	6	4	$v_3 = 0$
-4	400	200	500	?	
0	0	0	0	0	$v_4 = -6$
-2	-2	300	0	0	

$u_1 = 4 \quad u_2 = 4 \quad u_3 = 6 \quad u_4 = 6 \quad u_5 = 6$

Since  $\Delta_{ij} > 0$  at  $(3,5)$  the cell enters basic and  $(4,5)$  goes out of basis. Again calculating  $\Delta_{ij} = c_{ij} (u_i + v_j) - g_j$ .

	$v_j$					
$u_i$	5	8	6	6	3	800
4	-2	-5	-1	-1	-1	800
	400	7	7	6	5	$v_1 = -1$
8	-4	4	6	6	6	$v_2 = 0$
0	-2	0	0	0	0	$v_3 = 0$
	300	400	200	300	0	$v_4 = -6$
	800	400	200	200	200	$v_5 = 4$

Since all  $\Delta_{ij} \leq 0$ , optimality is obtained.

Optimum distribution is as follows:

A	5	B	6	6	3	800
P	600	7	7	6	5	
C	8	400	200	200	4	
D		E	F	G	H	

Minimum transportation cost:

$$= 3 \times 800 + 4 \times 600 + 6 \times 100 + 4 \times 400 + 6 \times 200 + 6 \times 200$$

$$= 9200.$$

- (a) (i) Prove that the group  $\frac{4\pi}{12\pi} = \mathbb{Z}_4$   
(ii) Show that  $\mathbb{Z}_4$  is not a homomorphic image of  $\mathbb{Z}_{12}$

(12)



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(b) Show that the function  $f$  defined on  $[0, 1]$  as

$f(x) = 2rx$ , if  $\frac{1}{r+1} < x < \frac{1}{r}$ ,  $r \in \mathbb{N}$ , is integrable over  $[0, 1]$  and

$$\int_0^1 f(x) dx = \frac{\pi^2}{6} \quad (13)$$

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- (d) Let  $x_1 = 2$ ,  $x_2 = 4$ , and  $x_3 = 1$  be a feasible solution to the system of equations

$$2x_1 + x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_2 - 18$$

Reduce the given feasible solution to a basic feasible solution.

(13)

P.T.O.



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R.T.O.

## SECTION-B

5. (a) Let  $G$  be a group and  $a, b \in G$ , such that  $a \neq b$  and  $O(a)$  and  $O(b)$  are relatively prime. Then Prove that  $O(ab) = O(a)O(b)$ . (10)

$$\text{Let } O(a) = m \quad O(b) = n$$

$$\begin{aligned} (ab)^{mn} &= a^{mn} b^{mn} \quad (\because ab = b \cdot a) \\ &= (a^m)^n (b^n)^m \\ &= e \end{aligned}$$

$$\Rightarrow O(ab) | mn \Rightarrow t | mn.$$

$$\Rightarrow mn = t \times .$$

$$\text{Now } a^t = a^{\frac{mn}{m}} = a^{\frac{mn}{n}} = (a^m)^{\frac{n}{n}} = e$$

$$b^t = b^{\frac{mn}{n}} = b^{\frac{mn}{m}} = (b^n)^{\frac{m}{m}} = e$$

$$\Rightarrow m | t \text{ and } n | t$$

$$\Rightarrow m, n | t \quad (\because m, n \text{ are relatively prime})$$

$$\Rightarrow mn = t$$

$$O8 \quad O(ab) = O(a)O(b).$$

(b) Show that  $Z[\sqrt{-7}]$  is not U.F.D.

(10)

(c) Examine the convergence of the integral

$$\int_2^3 \frac{dx}{(x-2)^{1/4}(3-x)^2} = \int_2^{2.5} \frac{dx}{(x-2)^{1/4}(3-x)^2} + \int_{2.5}^3 \frac{dx}{(x-2)^{1/4}(3-x)^2} \quad (10)$$

Convergence at  $x=2$  and  $x=3$  is doubtful because the integrand becomes  $\infty$  at these values.

Convergence at  $x=2$ :

Let  $f(x) = \frac{1}{(x-2)^{1/4}(3-x)^2}$  and  $g(x) = \frac{1}{(x-2)^{1/4}}$

$$\lim_{x \rightarrow 2^+} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 2^+} \frac{1}{(x-2)^2} = 1, \text{ which is nonzero finite}$$



So,  $\int_{2.5}^{2.5} \frac{1}{(x-2)^{1/4}(3-x)^2} dx$  converges as  $\int_{2.5}^{\infty} \frac{1}{(x-2)^{1/4}} dx$  converges.

Taking comparison integral  $\int_{2.5}^{\infty} \frac{1}{(3-x)^2} dx$  which

is divergent we observe that,

$$\lim_{x \rightarrow 3^-} \frac{f(x)}{g(x)} = 1 \text{ which is non zero finite.}$$

Hence  $\int_{2.5}^3 \frac{dx}{(2-x)^{1/4}(3-x)^2}$  is divergent so is the

Integral

$$\int_2^3 \frac{dx}{(3-x)^2(x-2)^{1/4}}$$

(i) If  $f(z) = \frac{z^2+5z+6}{z-2}$ , does Cauchy's theorem apply?

(10)

- (ii) When the path of integration C is a circle of radius 3 with origin as centre.
- (iii) When C is a circle (radius 1) with origin as centre.

Cauchy theorem: If  $f(z)$  is analytic in a domain D and continuous on a circle  $C$ , then

$$\oint f(z) dz = 0.$$

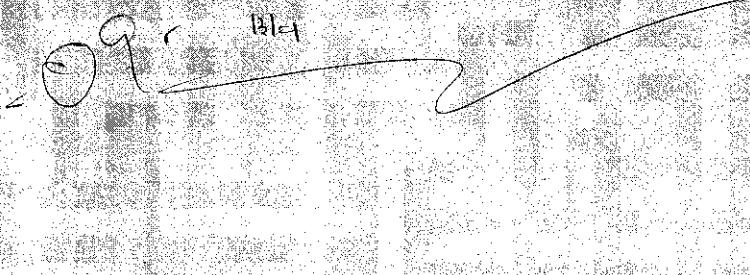
(i)  $C : |z|=3$ .

$f(z)$  has a singularity i.e., a pole at  $z=2$   
and hence not analytic. So Cauchy theorem  
does not apply.

(ii)  $f(z) = 1$  and continuous on  $|z|=1$

$f(z)$  is analytic within  $|z|=1$ . So Cauchy theorem applied

$$\int f(z) dz = 0$$



(c) Show that the following system of linear equations has degenerate solution

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

(10)

Degeneracy is obtained when any basic variable is at zero value.

Here no of basic variables is 2. Let

$(x_1, x_2)$  be basic variables

Assigning zero to  $x_3$  we get

$$2x_1 = 2 \Rightarrow x_1 = 1$$

$$3x_1 = 3$$

$\therefore (1, 0, 0)$  is a degenerate solution.

- Q. (ii) In the group  $S_3$ , show that the subset  
 $H = \{x \in S_3 : |x| \text{ divides } 2\}$  is not a subgroup. (10)

- (b) Let  $T$  denote the group of all non-singular upper triangular  $2 \times 2$  matrices with real entries i.e., the matrices of the

form,  $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$  where  $a, b, c \in \mathbb{R}$  and  $ac \neq 0$ . Show that  $H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \in T \right\}$  is a normal subgroup of  $T$ .

(10)



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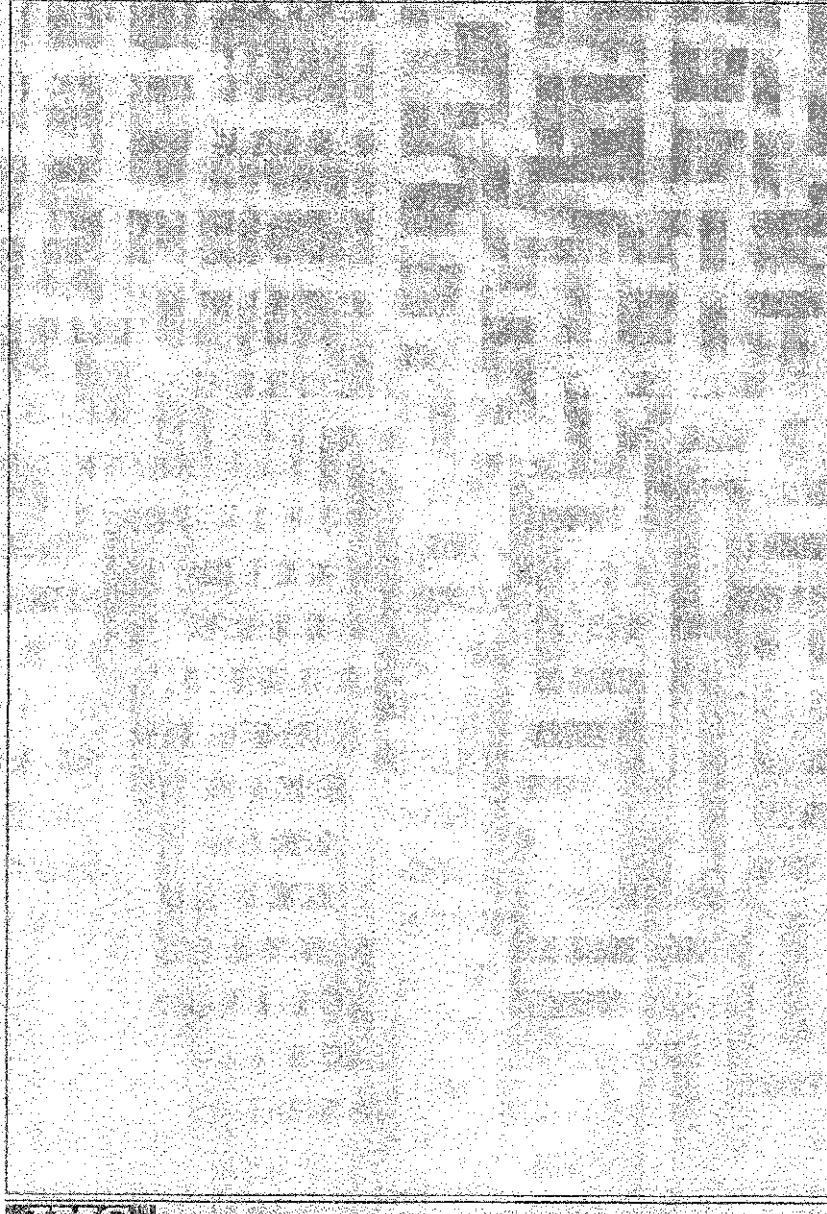
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P.T.O.

- (c) Let  $K$  be the set of all real numbers and  $R$  be the set of all real-valued continuous functions defined on  $R$ . Define  
 $(f+g)(x) = f(x)+g(x)$  and  $(fg)(x) = f(x)g(x)$   
for all  $f, g \in R$  and for all  $x \in R$ . Show that  $(R, +, \cdot)$  is a ring under the binary operations defined above.

(15)



LIVESTOCK

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P.T.O.

(3) Show that  $Z_1 \{z_1 + z_2 n, n \in \mathbb{Z}\} \subset \mathbb{C}^2$  is a Euclidean domain.

(15)

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PTO.

7. (a) Is the intersection of an arbitrary collection of open sets open? Justify your answer by a proof or by a counter example. (12)



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P.T.O.

(b) Discuss the convergence of the series.

$$x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots \quad (18)$$

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P.T.O.

(c) Find the maxima and minima of the function

$$f(x, y) = x^2 + y^2 - 3x - 12y + 20$$

(8)



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P.T.O.

- (d) Examine for uniform convergence and continuity of the limit function of the sequence  $\{f_n\}$ , where

$$f_n(x) = \frac{nx}{n^2 + x^2}, \quad 0 \leq x \leq 1. \quad (15)$$

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P.T.O.

8. (a) If  $a > c$ , use Rouche's theorem to prove that the equation  $e^z = az^3$  has no roots inside the circle  $|z| = 1$ .

(10)

(b) Find the Taylor's or Laurent's series which represent the function  $\frac{1}{(1+z^2)^{1/2}}$ . (15)

- (i) when  $|z| < 1$
- (ii) when  $1 < |z| < 2$
- (iii) when  $|z| > 2$

$$\text{Let } f(z) = \frac{1}{(1+z^2)(z+3)} = \frac{1}{5} \left[ \frac{1}{z+2} + \frac{2-z}{z^2+1} \right]$$

i) when  $|z| < 1$ , function is analytic throughout

$|z| < 1 \therefore$  Taylor's series for  $|z| < 1 \Rightarrow |z|^2 < 1$

$$\therefore \frac{|z|}{2} < 1$$

$$\begin{aligned} f(z) &= \frac{1}{5} \left( \frac{1}{2} \left( 1 + \frac{z}{2} \right)^{-1} + (2-z) \cdot (1+z^2)^{-1} \right) \\ &= \frac{1}{5} \left( \frac{1}{2} \left( 1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \dots \right) + (2-z) \left( 1 - z + z^2 - \dots \right) \right) \\ &\rightarrow \frac{1}{5} \left( \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{z}{2} \right)^n \right) + (2-z) \sum_{n=0}^{\infty} (-1)^n (z^2)^n \\ &= \frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \left( \frac{z}{2} \right)^n + \frac{(2-z)}{5} \sum_{n=0}^{\infty} (-1)^n z^{2n} \end{aligned}$$

ii) when  $1 < |z| < 2 \Rightarrow \frac{1}{|z|^2} < 1$  and  $\frac{|z|}{2} < 1$ .

Function is analytic in the above annular region

Expanding by Laurent's series we get,

$$f(z) = \frac{1}{5} \left( \frac{1}{2} \left( 1 + \frac{z}{2} \right)^{-1} + \frac{(2-z)}{z^2} \left( 1 + \frac{1}{z^2} \right)^{-1} \right)$$



$$= \frac{1}{5} \left( \frac{1}{2} \left( 1 - \frac{2}{z} + \frac{2^2}{z^2} - \dots \right) + \frac{(2-3)}{z^2} \left( 1 - \frac{1}{z} + \frac{1}{z^2} - \dots \right) \right)$$

$$= \frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \left( \frac{2}{z} \right)^n + \frac{(2-3)}{5 z^2} \sum_{n=0}^{\infty} (-1)^n \frac{1}{z^n}$$

iii) when  $|z| > 2 \Rightarrow \frac{|z|}{2} > 1 \text{ & } \frac{1}{|z|^2} < 1$

$$\Rightarrow \frac{2}{|z|} < 1$$

$$f(z) = \frac{1}{5} \left( \frac{1}{2} \left( 1 + \frac{2}{z} \right)^{-1} + \frac{(2-3)}{z^2} \left( 1 + \frac{1}{z} \right)^{-1} \right)$$

$$= \frac{1}{5} \left( \frac{1}{2} \left( 1 - \frac{2}{z} + \left( \frac{2}{z} \right)^2 - \dots \right) + \frac{(2-3)}{z^2} \left( 1 - \frac{1}{z} + \left( \frac{1}{z} \right)^2 - \dots \right) \right)$$

$$= \frac{1}{5 z^2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{2}{z} \right)^n + \frac{(2-3)}{5 z^2} \sum_{n=0}^{\infty} (-1)^n \left( \frac{1}{z} \right)^n$$

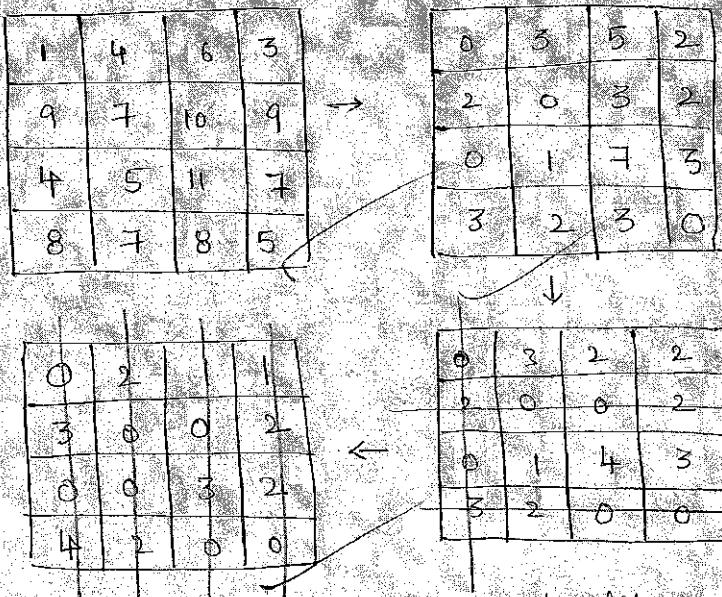
Hence found the required Taylor's and Laurent's series

(e) Solve the following assignment problem.

(10)

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	9	7
IV	8	7	8	5

Solving by Hungarian method.



∴ Number of lines is equal to 4, optimality is obtained.

0	2	1	1
3	X	0	2
X	0	3	2
4	2	X	0

The assignment is as follows:

$$\text{I} \rightarrow A$$

$$\text{II} \rightarrow C$$

$$\text{III} \rightarrow B$$

$$\text{IV} \rightarrow D$$

(d) Solve the following LP problem by simplex method.

(15)

$$\text{Minimize } z = 8x_1 - 2x_2$$

Subject to

$$-4x_1 + 2x_2 \leq 1$$

$$5x_1 - 4x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Let Objective function be  $\text{Max } z^* = -8x_1 + 2x_2$

subject to  $-4x_1 + 2x_2 \leq 1$

$$5x_1 - 4x_2 \leq 3 \quad x_1, x_2 \geq 0$$

Introducing slack variables  $s_1, s_2$  and  
converting into standard form, we get

$$-4x_1 + 2x_2 + s_1 + 0s_2 = 1$$

$$5x_1 - 4x_2 + 0s_1 + s_2 = 3$$

P.T.O.

$$\text{and } \text{Max } Z^* = -8x_1 + 2x_2 + 0s_1 + 0s_2$$

Number of variables is 4 and constraints is 2  
So making any (4-2) variables 0, i.e. let

$$x_1 = x_2 = 0 \text{ we get } s_1 = 1, s_2 = 3$$

It is a basic feasible solution, to obtain  
optimal basic feasible solution, proceeding to simplex table

	$C_j$	-8	2	0	0		
$C_B$	Base	$x_1$	$x_2$	$s_1$	$s_2$	b	0
0	$s_1$	-4	(2)	1	0	1	1/2
0	$s_2$	5	-4	0	1	3	-
$Z_f$	$\sum C_B C_{ij}$	0	0	0	0	0	
$C_j$	$C_j - Z_f$	-8	2	0	0		

$x_2$  is increasing and  $s_1$  is outgoing variable

	$C_j$	-8	2	0	0		
$C_B$	Base	$x_1$	$x_2$	$s_1$	$s_2$	b	0
2	$x_2$	-2	1	1/2	0	4	
0	$s_2$	-3	0	2	1	5	
$Z_f$		-4	2	1	0	1	
$C_j$		4	0	-1	0		

$$\text{and } \text{Max } Z^* = -8x_1 + 2x_2 + 0s_1 + 0s_2$$

Number of variables is 4 and constraints is 2  
So making any (4-2) variables zero, i.e., let

$$x_1 = x_2 = 0 \text{ we get } s_1 = 1, s_2 = 3$$

It is a basic feasible solution, to obtain  
optimal basic feasible solution, proceeding to simplex table.

$C_B$	$C_j$	-8	2	0	0		
$C_B$	Basic	$x_1$	$x_2$	$s_1$	$s_2$	b	0
0	$s_1$	-4	(2)	1	0	1	1/2
0	$s_2$	5	-4	0	1	3	-
$Z_f = \sum C_B U_j$		0	0	0	0	0	
$C_B$	$C_j - Z_f$	-8	2	0	0		

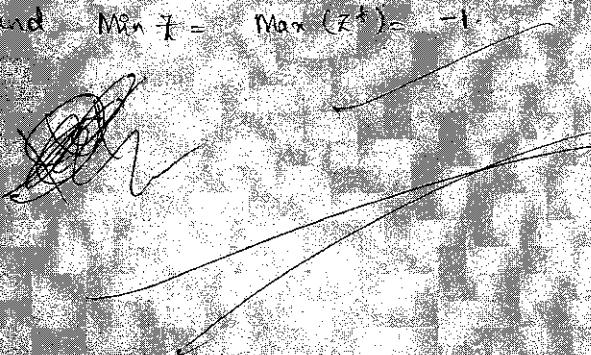
$(x_2)$  is incoming and  $s_1$  is outgoing variable

$C_B$	$C_j$	-8	2	0	0		
$C_B$	Basic	$x_1$	$x_2$	$s_1$	$s_2$	b	0
2	$x_2$	-2	1	1/2	0	$y_2$	
0	$s_2$	-3	0	2	1	5	
$Z_f$		-4	2	1	0	1	
$C_B$	$C_j - Z_f$	4	0	-1	0		

Optimality is obtained. Solution is degenerate, because more than 2 variables are zero.

solution is  $(x_1, x_2) = (0, 1/2)$

$$\text{and } \min \bar{Y} = \max (Z^+) = -1.$$



# ROUGH SPACE

$$\left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array}\right) \left(\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}\right) = \left(\begin{array}{cc} 1 & 2 \\ 0 & 1 \end{array}\right) \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array}\right) = \left(\begin{array}{cc} 1 & 3 \\ 0 & 1 \end{array}\right) \quad 2 \times x = \frac{1}{x+1} < x < \frac{1}{\frac{1}{x}}$$

$$\begin{aligned} & \text{Left side: } \int_{\gamma_1}^{\gamma_2} f(x) dx = \int_{\gamma_1}^{\gamma_2} \left( \frac{1}{2} x^2 - \frac{1}{3} x^3 \right) dx \\ & \quad = \left[ \frac{1}{2} \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^4}{4} \right]_{\gamma_1}^{\gamma_2} = \frac{1}{6} \gamma_2^3 - \frac{1}{12} \gamma_2^4 - \left( \frac{1}{6} \gamma_1^3 - \frac{1}{12} \gamma_1^4 \right) \\ & \quad = \frac{1}{6} (\gamma_2^3 - \gamma_1^3) - \frac{1}{12} (\gamma_2^4 - \gamma_1^4) \\ & \quad = \frac{1}{6} (\gamma_2 - \gamma_1)(\gamma_2^2 + \gamma_1 \gamma_2 + \gamma_1^2) - \frac{1}{12} (\gamma_2^3 - \gamma_1^3) \\ & \quad = \frac{1}{6} (\gamma_2 - \gamma_1) \left( \gamma_2^2 + \gamma_1 \gamma_2 + \gamma_1^2 - \frac{1}{2} (\gamma_2^2 + \gamma_1 \gamma_2 + \gamma_1^2) \right) \\ & \quad = \frac{1}{6} (\gamma_2 - \gamma_1) \left( \frac{1}{2} (\gamma_2^2 + \gamma_1 \gamma_2 + \gamma_1^2) \right) \end{aligned}$$

$$f(n) = n! \cdot (1 - \frac{1}{n})^n$$

$$f'(n) = n! \left( \left(1 - \frac{1}{n}\right)^{n-1} \right) + n! \cdot \left( \ln\left(1 - \frac{1}{n}\right)\right) \left(1 - \frac{1}{n}\right)^{n-2} \cdot \left(-\frac{1}{n^2}\right) = 0$$