

online

181/250



A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

TEST SERIES (MAIN)-2014

Test Code: FULL TEST P-II(M) IAS / T-08

MATHEMATICS

by K. VENKANNA

The person with 14 years of Teaching Experience

FULL TEST P-II

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has 52 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/nnotations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name: Nitish.k
Roll No.: 149709
Test Centre: Bangalore
Medium:

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them.
Signature of the Candidate: Nitish.k

I have verified the information filled by the candidate above.
Signature of the invigilator:

IMPORTANT NOTE: Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			08
	(b)			08
	(c)			07
	(d)			08
	(e)			08
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			08
	(b)			11
	(c)			10
	(d)			11
5	(a)			08
	(b)			08
	(c)			08
	(d)			09
	(e)			<del>08</del>
6	(a)			10
	(b)			09
	(c)			04
	(d)			13
7	(a)			22
	(b)			08
	(c)			08
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				181/250

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39

181/250

## SECTION-A

1. (a) Suppose that  $a$  and  $b$  are group elements. If  $|b| = 2$  and  $bab = a^4$ , determine the possibilities for  $|a|$ . (10)

$$b^2 = e \Rightarrow b = b^{-1}$$

$$\therefore a^4 = bab = b^{-1}ab$$

$$a^8 = b^{-1}ab \cdot b^{-1}ab = b^{-1}a^2b$$

$$a^{16} = b^{-1}a^4b = b^{-1}(bab)b$$

$$a^{16} = (b^{-1}b)a(bb) = a$$

$$\Rightarrow \boxed{a^{15} = e}$$

$$\Rightarrow o(a) \mid 15$$

$$\Rightarrow \underline{\underline{o(a) = 1, 3, 5 \text{ or } 15}}$$

1. (b) Let  $R$  be a ring and let  $M_2(R)$  be the ring of  $2 \times 2$  matrices with entries from  $R$ . Explain why these two rings have the same characteristic. (10)

Let  $\text{char}(R) = n$ .

$\Rightarrow na = 0 \forall a \in R$  where  $n$  is the least positive integer.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$  where  $a, b, c, d \in R$

$\Rightarrow na = 0; nb = 0; nc = 0; nd = 0$   
where again  $n$  is the least positive integer.

$n \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

and  $n$  is the least positive integer.

$\therefore \text{char}(M_2(R)) = n$

$\text{char}(R) = \text{char}(M_2(R))$

1. (d) If  $u+v = \frac{2\sin 2x}{e^{2x} + e^{-2x} - 2\cos 2x}$ , and  $f(z) = u+iv$  is an analytic function of  $z = x+iy$ , find  $f(z)$  in terms of  $z$ . (10)

Using Thompson-Milne method  $\Rightarrow x = z$  &  $y = 0$

$$u_x + v_x = 2 \left[ \frac{(2 - 2\cos 2z)(2\cos 2z) - 4\sin^2 2z}{(2 - 2\cos 2z)^2} \right] \quad \text{--- (1)}$$

$$u_y + v_y = 2 \left[ \frac{-\sin 2z}{(2 - 2\cos 2z)^2} \right] \times (2 - 2) = 0 \quad \text{--- (2)}$$

by C.R equations  $u_x = v_y$  &  $v_x = -u_y$

$\therefore$  (2) becomes  $\Rightarrow u_x - v_x = 0 \Rightarrow \boxed{u_x = v_x}$

$\therefore$  (1) become  $u_x = \frac{(1 - \cos 2z)(\cos 2z) - \sin^2 2z}{(1 - \cos 2z)^2}$

$$u_x = \frac{\cos 2z - (\cos^2 2z + \sin^2 2z)}{(1 - \cos 2z)^2} = \frac{\cos 2z - 1}{(1 - \cos 2z)^2}$$

$$\therefore u_x = \frac{1}{1 - \cos 2z} = \frac{-1}{2\sin^2 z} = -\frac{1}{2} \operatorname{cosec}^2 z.$$

$$\therefore f'(z) = u_x + iv_x = (1+i)u_x = -\frac{1}{2}(1+i) \operatorname{cosec}^2 z$$

$$\int \Rightarrow f(z) = -\frac{1}{2}(1+i) \int \operatorname{cosec}^2 z \, dz + c$$

$$\boxed{f(z) = \frac{1}{2}(1+i) \cot z + c}$$

consider  $I = \int_0^{\infty} f(x) dx$  where  $f(x) = \frac{e^x - (1+x)}{(1+x)e^x}$

why  $\lim_{x \rightarrow 0} f(x) = 0$   $\therefore 0$  is not a point of infinite discontinuity

$\int_0^1 f(x) dx + \int_1^{\infty} f(x) dx$  where first integral is proper integral.

consider  $g(x) = \frac{1}{x^2}$

$$\frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \left\{ \frac{e^x - (1+x)}{e^x} \cdot \frac{x}{1+x} \right\}$$

$$\lim_{x \rightarrow \infty} \frac{e^x - (1+x)}{e^x} \cdot \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{x} + 1}$$

$$\lim_{x \rightarrow \infty} \frac{e^x - 1}{e^x} \cdot \frac{1}{0 + 1} \quad \left( \frac{\infty}{\infty} \text{ form} \right)$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{e^x} = 1 // \text{ (neither zero nor infinity)}$$

$\int_0^1 f(x) dx$  &  $\int_1^{\infty} g(x) dx$  behave identically

Efficiency Well

	W <sub>1</sub>	W <sub>2</sub>	W <sub>3</sub>	W <sub>4</sub>	W <sub>5</sub>
P <sub>1</sub>	45	40	65	30	55
P <sub>2</sub>	50	30	25	60	30
P <sub>3</sub>	25	20	15	20	40
P <sub>4</sub>	35	25	30	25	20
P <sub>5</sub>	80	60	60	70	50

This is a maximi problem.

Subtract each elem by the largest elem i.e. 80

40	15	50	25
50	55	20	50
60	65	60	40
55	50	55	60
20	20	10	30

20	25	<del>0</del>	35	10
10	30	35	0	30
15	20	25	20	0
0	10	<del>15</del>	10	15
0	20	20	10	30

W <sub>2</sub>	W <sub>3</sub>	W <sub>4</sub>	W <sub>5</sub>
15	0	35	10
20	35	0	30
10	25	20	0
0	5	10	15
10	20	10	30

number of assignments = 5 = no. of rows

or the optimal assignment is

- W<sub>3</sub>
- W<sub>4</sub>

.....

$ab = 0$  implies  $a = 0$  or  $b = 0$ .

$| ab = ac$  and  $a \neq 0$  imply  $b = c$ .

the  $n$  you found prime?

$n = 6$  so that we are in the ring  $Z_6$ .

$$3^2 = 9 = 3 \Rightarrow 3^2 = 3$$

$$3 \neq 0 \text{ not } 3 \neq 1$$

$$2 = 6 = 0 \Rightarrow 3 \neq 0 \text{ or } 2 \neq 0$$

$$3 \cdot 5 = 15 = 3 \Rightarrow 3 \cdot 5 = 3 \cdot 3$$

$$3 \cdot 3 = 9 = 3 \text{ but } 3 \neq 3$$

~~3 is not a prime.~~  
 $n$  is not a prime.

$n$  is prime then  $Z_n$  is a field and all the above conditions would be satisfied.

4. (b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that  $f$  is differentiable on  $\mathbb{R}$  but  $f'$  is not continuous on  $\mathbb{R}$ .

(13)

$$R f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} h \sin\left(\frac{1}{h}\right) = 0 \times \mathbb{R} = 0$$

where  $\mathbb{R}$  oscillates finitely b/w  $-1$  &  $1$

$$L f'(0) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} -h \sin\left(\frac{1}{h}\right)$$

$$= 0 \times \mathbb{R} = 0$$

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P.T.O.

$$x \neq 0 \Rightarrow f'(x) = 2x \sin\left(\frac{1}{x^2}\right) - \frac{2}{x} \cos\left(\frac{1}{x^2}\right)$$

is differentiable  $\forall x \neq 0$ . — (2)

m (1) & (2)  $\Rightarrow f$  is differentiable on  $\mathbb{R}$ .

$$f' = \begin{cases} 2x \sin\left(\frac{1}{x^2}\right) - \frac{2}{x} \cos\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$f'(x)$  does not exist as  $\frac{1}{x} \cos\left(\frac{1}{x^2}\right)$ .

$f'$  is not continuous at origin and is discontinuity of second kind.

4. (c) Expand  $f(z) = \frac{z+3}{z(z^2-z-2)}$  in powers of  $z$ ;  
 where (i)  $|z| < 1$ ,  
 (ii)  $|z| < 2$ , (iii)  $|z| > 2$ .

$$f(z) = \frac{-3}{2z} + \frac{5}{6} \frac{1}{z-2} + \frac{2}{3} \frac{1}{z+1}$$

①  $|z| < 1$

$$\begin{aligned} f(z) &= \frac{-3}{2z} - \frac{5}{12} \left[1 - \frac{z}{2}\right]^{-1} + \frac{2}{3} [1+z]^{-1} \\ &= \frac{-3}{2z} - \frac{5}{12} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \frac{2}{3} \sum_{n=0}^{\infty} (-1)^n z^n \end{aligned}$$

②  $1 < |z| < 2 \Rightarrow \frac{1}{|z|} < 1$  and  $\frac{|z|}{2} < 1$

$$\begin{aligned} f(z) &= \frac{-3}{2z} - \frac{5}{12} \left[1 - \frac{z}{2}\right]^{-1} + \frac{2}{3z} \left[1 + \frac{1}{z}\right]^{-1} \\ f(z) &= \frac{-3}{2z} - \frac{5}{12} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \frac{2}{3z} \sum_{n=0}^{\infty} (-1)^n \frac{1}{z^n} \end{aligned}$$

③  $|z| > 2 \Rightarrow \frac{2}{|z|} < 1$

$$\begin{aligned} f(z) &= \frac{-3}{2z} + \frac{5}{6z} \left[1 - \frac{2}{z}\right]^{-1} + \frac{2}{3z} \left[1 + \frac{1}{z}\right]^{-1} \\ &= \frac{-3}{2z} + \frac{5}{6z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n + \frac{2}{3z} \sum_{n=0}^{\infty} (-1)^n \frac{1}{z^n} \end{aligned}$$

Make a graphical representation of the set of constraints of the following LPP. Find extreme points of the feasible region. Finally, solve the problem graphically.

Maximise  $Z = 2x_1 + x_2$

subject to  $x_1 + x_2 \geq 5$

$$2x_1 + 3x_2 \leq 20$$

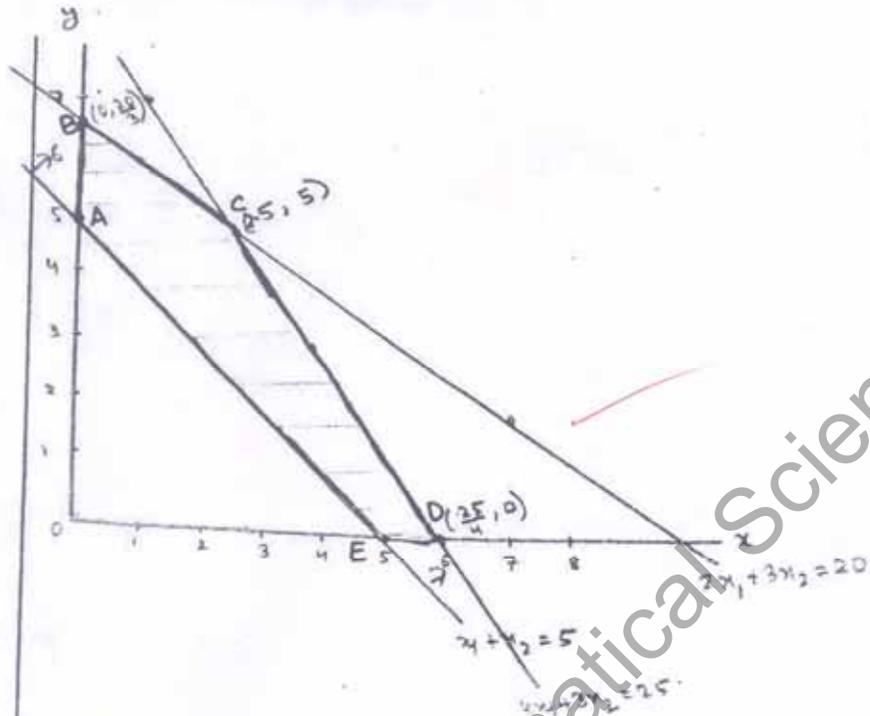
$$4x_1 + 3x_2 \leq 25$$

$$x_1, x_2 \geq 0.$$

$$x_1 + x_2 = 5 \Rightarrow (0, 5) \text{ \& } (5, 0)$$

$$-3x_2 = 20 - 2x_1 \Rightarrow (1, 6) \text{ \& } (7, 2)$$

$$3x_2 = 25 - 4x_1 \Rightarrow (1, 7) \text{ \& } (4, 3)$$



Extreme point .

$$A = (0, 5) \Rightarrow z = 5$$

$$B = (0, \frac{20}{3}) \Rightarrow z = 20/3 = 6.67$$

$$C = (2.5, 5) \Rightarrow z = 10$$

$$D = (\frac{25}{4}, 0) \Rightarrow z = 25/2 = 12.5$$

$$E = (5, 0) \Rightarrow z = 10$$

$$z_{\max} = 12.5$$

$$x_1 = \frac{25}{4} \text{ \& } x_2 = 0$$

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{y}{z} \frac{\partial z}{\partial y}\right)^2 = 1.$$

$$\frac{1}{x} dx = dX; \quad \frac{1}{y} dy = dY; \quad \frac{1}{z} dz = dZ$$

$$\log x = X; \quad \log y = Y; \quad \log z = Z.$$

$$\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 = 1$$

$$P^2 + Q^2 = 1 \quad \text{where } P = \frac{\partial Z}{\partial X} \text{ and } Q = \frac{\partial Z}{\partial Y}$$

$$z = aX + bY + c \quad \text{where } a^2 + b^2 = 1$$

$$z = a \log x + \sqrt{1-a^2} \log y + c'$$

$$= c' \log \frac{x^a y^{\sqrt{1-a^2}}}{y}$$

$$\text{where } c' = \log c.$$

5. (b) Find a surface satisfying  $r - 2s + t = 6$  and touching the hyperbolic paraboloid  $z = xy$  along its section by the plane  $y = x$ . (1)

$$(D^2 - 2DD' + D'^2)z = 6$$

$$\Rightarrow (D - D')^2 z = 6 \quad m = 1, 1.$$

$$C.F = \phi_1(y+x) + x \phi_2(y+x)$$

$$P.I = \frac{1}{(D - D')^2} \cdot 6 = \frac{1}{D^2} \left[ 1 - \frac{D'}{D} \right]^{-2} \cdot 6$$

$$= \frac{1}{D^2} \left[ 1 + 2 \frac{D'}{D} \right] 6 = \frac{1}{D^2} \cdot 6 = 6 \cdot \frac{x^2}{2}$$

$$= 3x^2. \quad \checkmark$$

$$\therefore z = \phi_1(y+x) + x\phi_2(y+x) + 3x^2 \quad \left. \begin{array}{l} \text{touch} \\ \text{along } y=x \end{array} \right\}$$

$$z = xy.$$

$\therefore p$  &  $q$  values are equal.

$$\left. \begin{array}{l} \phi_1'(y+x) + x\phi_2'(y+x) + \phi_2(y+x) + 6x = y \\ \phi_1'(y+x) + x\phi_2'(y+x) = x \end{array} \right\} \begin{array}{l} \text{put} \\ x=y \end{array}$$

$$\Rightarrow \phi_1'(2x) + x\phi_2'(2x) + \phi_2(2x) + 5x = 0 \quad \text{--- (1)}$$

$$\phi_1'(2x) + x\phi_2'(2x) = x \quad \text{--- (2)}$$

$$\text{using (2) in (1)} \Rightarrow \phi_2(2x) = -6x = -3(2x)$$

$$\Rightarrow \phi_2(x) = -3x \Rightarrow \phi_2'(x) = -3.$$

$$\text{Putting in (2)} \Rightarrow \phi_1'(2x) - 3x = x$$

$$\phi_1'(2x) = 4x = 2(2x) \Rightarrow \phi_1'(x) = 2x$$

$$\phi_1(x) = x^2 + C.$$

$$z = (y+x)^2 + C + (-3x)(y+x) + 3x^2 \quad \left. \begin{array}{l} \text{equating} \\ \& \text{put } x=y \end{array} \right\}$$

$$z = xy.$$

$$\text{we get } C = 0$$

$$\therefore z = x^2 + y^2 - xy$$

5. (c) The current  $i$  in an electric circuit is given by  $i = 10e^{-t} \sin 2\pi t$  where  $t$  is in seconds. Using Newton's method, find the value of  $t$  correct to 3 decimal places for  $i = 2$  amp.

$$t_{n+1} = t_n - \frac{i(t_n)}{i'(t_n)}$$

Putting  $i = 2$ .

$$10e^{-t} \sin 2\pi t - 2 = 0$$

$$\text{let } f(t) = 10e^{-t} \sin 2\pi t - 2$$

$$t_{n+1} = t_n - \frac{f(t_n)}{f'(t_n)}$$

$$\begin{aligned} f'(t) &= 10 \left[ -e^{-t} \sin 2\pi t + 2\pi e^{-t} \cos 2\pi t \right] \\ &= 10e^{-t} \left[ 2\pi \cos 2\pi t - \sin 2\pi t \right] \end{aligned}$$

starting with  $t_0 = 0$

$$t_1 = 0.03183$$

$$t_2 = 0.03314$$

$$t_3 = 0.03314$$

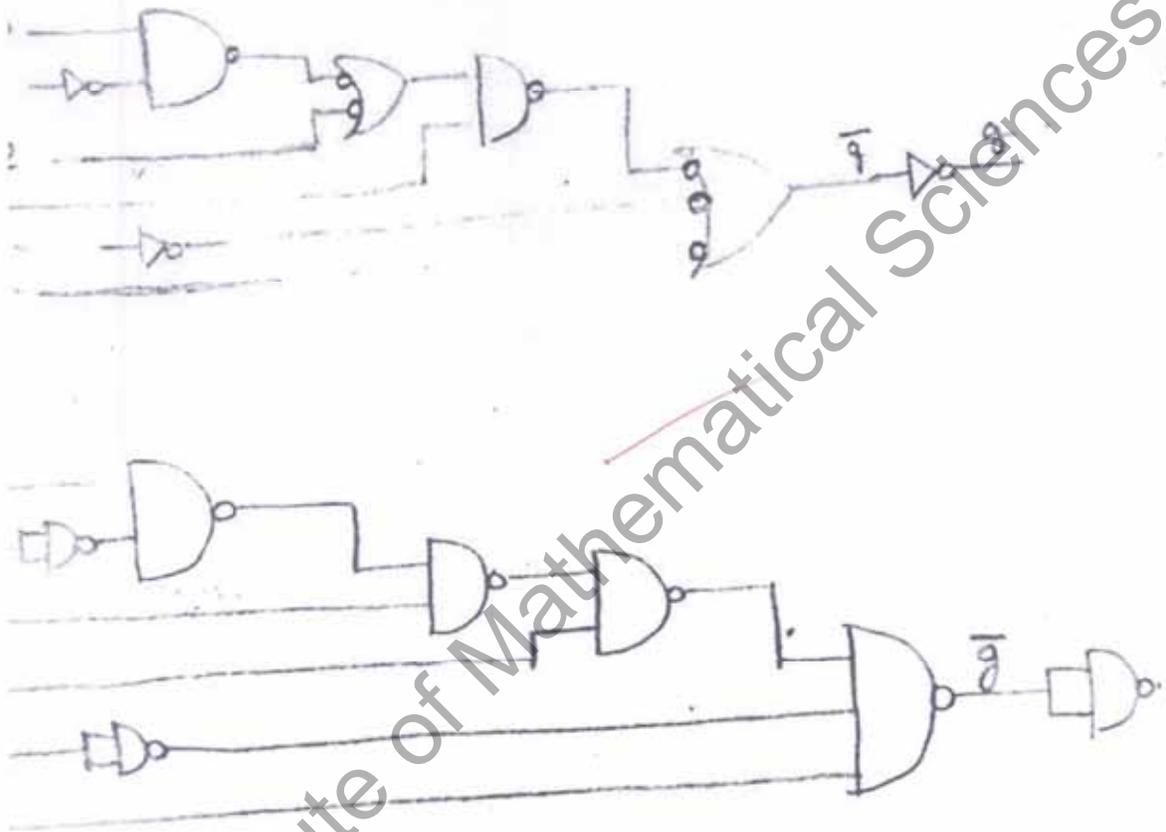
$$t_4 = 0.03314$$

$$t = 0.03314 \text{ seconds.}$$

where  $\bar{x}$  denotes the complement of  $x$ .

i) Find the decimal equivalent of  $(357.32)_8$

$$j = ab\bar{c} + d + a\bar{e} + \bar{f} = a(b\bar{c} + \bar{e}) + d + \bar{f}$$



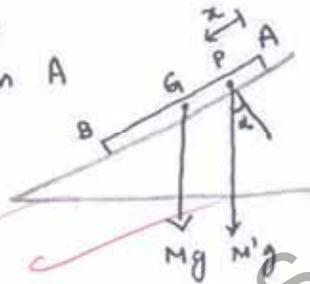
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5. (c) A plank of mass  $M$  is initially at rest along a line of greatest slope of a smooth plane inclined at an angle  $\alpha$  to the horizon, and a man of mass  $M'$ , starting from the upper end, walks down the plank so that it does not move, show that he gets to the other end in

$$\sqrt{\frac{2M'a}{(M+M')g \sin \alpha}}, \text{ where } a \text{ is the length of the plane.}$$

let  $\bar{x}$  be distance of centre of gravity from A



$$\Rightarrow (M+M')\bar{x} = M \frac{a}{2} + M'x$$

$$\Rightarrow \bar{x} = \frac{M'x + M \frac{a}{2}}{M+M'}$$

$$\Rightarrow \ddot{\bar{x}} = \frac{M' \ddot{x}}{M+M'} \Rightarrow \ddot{x}$$

~~⊙~~  
No marks please

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6. (a) Solve  $(D+D'-1)(D+D'-3)(D+D')z = e^{x+y} \sin(2x+y)$ 

(12)

Complementary function

$$C.F = e^x \phi_1(y-x) + e^{3x} \phi_2(y-x) + \phi_3(y-x)$$

Particular integral

$$P.I = \frac{1}{f(D,D')} e^{x+y} \sin(2x+y) = e^{x+y} \cdot \frac{1}{f(D+1, D'+1)} \sin(2x+y)$$

$$= e^{x+y} \frac{1}{(D+D'+2)} \frac{1}{D^2+2DD'+D'^2-1} \sin(2x+y)$$

$$\begin{aligned} D^2 &\rightarrow -4 \\ D^2 &\rightarrow -1 \\ DD' &\rightarrow -2 \end{aligned}$$

$$= \frac{e^{x+y}}{(D+D'+2)} \frac{1}{-4-4-1-1} \sin(2x+y)$$

$$= -\frac{1}{10} e^{x+y} \frac{1}{D+D'+2} \sin(2x+y)$$

$$= -\frac{1}{10} e^{x+y} \frac{(D+D'-2) \sin(2x+y)}{(D+D')^2-4}$$

$$= -\frac{1}{10} e^{x+y} \frac{(D+D'-2) \sin(2x+y)}{D^2+2DD'+(D')^2-4}$$

$$= \frac{1}{130} e^{x+y} [2 \cos(2x+y) + \cos(2x+y) - 2 \sin(2x+y)]$$

$$\xi = \phi(x, y) = y/x \quad ; \quad \eta = \psi(x, y) = x$$

$$\phi_x = -y/x^2 \quad ; \quad \phi_y = 1/x \quad ; \quad \phi_{xx} = \frac{2y}{x^3} \quad ; \quad \phi_{yy} = 0 \quad ; \quad \phi_{xy} = -1/x^2$$

$$\psi_x = 1 \quad ; \quad \psi_y = 0 \quad ; \quad \psi_{xx} = 0 = \psi_{xy} = \psi_{yy}$$

$$2 \left( \frac{y^2}{x^4} \right) + 2xy \left( -\frac{y}{x^3} \right) + y^2 \left( \frac{1}{x^2} \right) = \frac{2y^2}{x^2} - \frac{2y^2}{x^2} = 0$$

$$2(1) + 2xy(0) + y^2(0) = 2$$

$$x^2 \left( -\frac{y}{x^2} \right) + 2xy \left( 0 + \frac{1}{x} \right) + 2y^2(0) = 0$$

$$x^2 \left( \frac{2y}{x^3} \right) + 2xy \left( -\frac{1}{x^2} \right) + y^2(0) = \frac{2y}{x} - \frac{2y}{x} = 0$$

$$(x^2(0) + 0 + 0) u_{\eta\eta} = 0$$

Canonical form  $\Rightarrow Au_{\xi\xi} + Bu_{\xi\eta} + Cu_{\eta\eta}$

$(2) u_{\eta\eta} = 0 \Rightarrow \boxed{u_{\eta\eta} = 0}$  is the Canonical form.

$$\eta = f(\xi)$$

$\dots (x_0)$

$$\therefore z = e^x \phi_1(y-x) + e^{3x} \phi_2(y-x) + \phi_3(y-x) + \frac{1}{130} e^{x+y} [3 \cos(2x+y) - 2 \sin(2x+y)]$$

6. (b) Reduce  $x^2r+2xys+y^2t=0$  to canonical form and hence solve it. (1)

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$$

Comparing with  $au_{xx} + bu_{xy} + cu_{yy} = 0$

$$a = x^2; b = 2xy; c = y^2$$

characteristic eqn

$$x^2 \lambda^2 + 2xy \lambda + y^2 = 0$$

$$\Rightarrow (x\lambda + y)^2 = 0 \Rightarrow x\lambda + y = 0$$

$$\Rightarrow \lambda = -y/x$$

$$\therefore \frac{dy}{dx} + \lambda = 0 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 0 \Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$$

$$\Rightarrow \log\left(\frac{y}{x}\right) = \log c$$

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P.T.O

(c) The following table gives the velocity  $v$  of a particle at time  $t$  :

$t$  (seconds): 0 2 4 6 8 10 12 (10)

$v$  (m/sec): 4 6 16 34 60 94 136  
 $v_0$   $v_1$   $v_2$   $v_3$   $v_4$   $v_5$   $v_6$

$$S = \int v dt \quad n = 6$$

$$h = 2$$

Applying Simpson's  $\frac{1}{3}$ rd rule

$$S = \frac{h}{3} \left[ (v_0 + v_6) + 4(v_1 + v_3 + v_5) + 2(v_2 + v_4) \right]$$

$$= \frac{2}{3} \left[ (4 + 136) + 4(6 + 34 + 94) + 2(16 + 60) \right]$$

04

$$= \underline{\underline{552 \text{ m}}}$$

ng in matrix form  $AX = B$ .

$$\begin{bmatrix} 27 & 6 & -1 \\ 6 & 15 & 2 \\ 1 & 1 & 54 \end{bmatrix}; B = \begin{bmatrix} 85 \\ 140 \\ 72 \end{bmatrix}$$

ss-Seidal in Matrix form

$$x^{k+1} = -(D+L)^{-1} U x^k + (D+L)^{-1} B$$

$$\Rightarrow D = \begin{bmatrix} 27 & 0 & 0 \\ 6 & 15 & 0 \\ 1 & 1 & 54 \end{bmatrix}; U = \begin{bmatrix} 0 & 6 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 0.037 & 0 & 0 \\ -0.014 & 0.0666 & 0 \\ -4.115 \times 10^{-4} & -1.234 \times 10^{-3} & 0.018 \end{bmatrix}$$

$$x_0 = [0, 0, 0]$$

$$x_1 = \begin{bmatrix} 3.148 \\ 3.5407 \end{bmatrix}; x_2 = \begin{bmatrix} 2.4321 \\ 3.572 \\ 1.9258 \end{bmatrix}$$

$$\begin{cases} x = 2.4254 \\ y = 3.573 \\ z = 1.926 \end{cases}$$

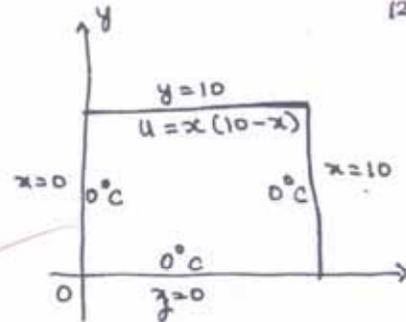
7. (a) A square plate is bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x = 10$  and  $y = 10$ . Its faces are insulated. The temperature along the upper horizontal edge is given by  $u(x, 10) = x(10 - x)$  while the other three faces are kept at  $0^\circ\text{C}$ . Find the steady state temperature in the plate. [2]

$$u(x, 0) = 0 \quad \text{--- (4)}$$

$$u(0, y) = 0 \quad \text{--- (5)}$$

$$u(10, y) = 0 \quad \text{--- (6)}$$

$$u(x, 10) = x(10 - x) \quad \text{--- (7)}$$



The possible solutions are.

$$u = (Ae^{px} + Be^{-px})(C \cos py + D \sin py) \quad \text{--- (1)}$$

$$u = (A \cos px + B \sin px)(C e^{py} + D e^{-py}) \quad \text{--- (2)}$$

$$u = (Ax + B)(Cy + D) \quad \text{--- (3)}$$

2 possible solution.

$$y) = (A \cos px + B \sin px)(C e^{py} + D e^{-py})$$

$$g \text{ (5)} \Rightarrow \underline{0 = A}$$

$$g \text{ (6)} \Rightarrow B \sin 10p = 0 \Rightarrow \sin 10p = 0 \\ \Rightarrow 10p = n\pi \Rightarrow \boxed{p = \frac{n\pi}{10}}; n = 1, 2, \dots$$

$$g \text{ (4)} \Rightarrow C + D = 0 \Rightarrow \underline{D = -C}$$

$$x, y) = 2BC \sin px \sinh(py)$$

$$x, y) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{10} x\right) \sinh\left(\frac{n\pi}{10} y\right)$$

$$y) = \sum_{n=1}^{\infty} \{b_n \sinh(n\pi)\} \sin\left(\frac{n\pi}{10} x\right)$$

$$-x^2 = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{10} x\right)$$

7. (b) Using modified Euler's method, find an approximate value of  $y$  when  $x=0.3$ , given that  $dy/dx = x + y$  and  $y=1$  when  $x=0$ . (10)

$$f(x, y) = x + y; \quad x_0 = 0; \quad y_0 = 1; \quad h = 0.3; \quad x_1 = 0.3$$

$$y_1 = y(x_1) = y(0.3) = ?$$

$$\begin{aligned} y_1^{(0)} &= y_0 + h f(x_0, y_0) \\ &= 1 + 0.3(0 + 1) = 1.3 \end{aligned}$$

$$\begin{aligned} y_1^{(1)} &= y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})] \\ &= 1 + \frac{0.3}{2} [1 + 0.3 + y_1^{(0)}] = 1.39 \end{aligned}$$

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})] = 1.4035$$

$$y_1^{(3)} = 1.4055$$

$$y_1^{(4)} = 1.4058$$

$$y_1^{(5)} = 1.4058$$

$$\therefore \underline{y_1 = y(x_1) = y(0.3) = 1.4058}$$

$$\begin{aligned}
 B_n &= \frac{2}{10} \int_0^{10} (10x - x^2) \sin\left(\frac{n\pi}{10} x\right) dx \\
 &= \frac{2}{10} \left[ (10x - x^2) \left(-\frac{\cos kx}{k}\right) - (10 - 2x) \left(-\frac{\sin kx}{k^2}\right) \right. \\
 &\quad \left. + (-2) \left(\frac{\cos kx}{k^3}\right) \right]_0^{10} \quad \text{where } k = \frac{n\pi}{10} \\
 &= \frac{2}{10} \left[ (-10) \frac{\sin 10k}{k^2} - 2 \frac{\cos 10k}{k^3} - \left(-\frac{2}{k^3}\right) \right] \\
 &= \frac{2}{10} \left[ \frac{-2 \cos n\pi + 2}{k^3} \right] \quad \text{where } k = \frac{n\pi}{10} \\
 &= \frac{4}{10k^3} (1 - \cos n\pi) = \begin{cases} 0, & n \text{ is even.} \\ \frac{8}{10k^3}, & n \text{ is odd.} \end{cases}
 \end{aligned}$$

$$\Rightarrow b_n = \frac{8}{10k^3 \sinh(n\pi)} \quad \text{where } n \text{ is odd.}$$

$$\therefore u(x, y) = \frac{800}{\pi^3} \sum_{m=1}^{\infty} \frac{1}{(2m-1)^3} \frac{1}{\sinh[(2m-1)\pi]}$$

$$\times \sin\left(\frac{(2m-1)\pi}{10} x\right) \sinh\left(\frac{(2m-1)\pi}{10} y\right)$$

is the required solution.

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7. (c) For the given set of data points  
 $(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_n, f(x_n))$   
write an algorithm to find the value of  $f(x)$  by using Lagrange's interpolation formula. (1)

step 1: Enter  $x$  and  $f(x)$  for  $i = 1$  to  $n$

step 2: sum = 0

step 3: for  $i = 1$  to  $n$  in steps of 1

step 4: prod = 1

step 5: for  $j = 1$  to  $n$  in steps of 1

step 6: if  $i = j$   
then  $j = j + 1$

step 7:  $prod = prod \times \frac{x - x_j}{x_i - x_j}$

step 8: end for

step 9:  $sum = sum + prod \cdot f(x_i)$

step 10: end for

step 11: print "sum"

8. (a) Determine the motion of a spherical pendulum, by using Hamilton's equations.

(16)