

180/250

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

R&R



TEST SERIES (MAIN)-2014

Test Code: S-AUG-14/IAS(M) / Test - 2

MATHEMATICS

by K. VENKANNA

The person with 14 years of Teaching Experience

PAPER-II: Algebra, Complex Analysis, Real Analysis & LPP

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

- This question paper-cum-answer booklet has **56** pages and has **42 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- Symbols/notations carry their usual meanings, unless otherwise indicated.
- All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

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ENGLISH

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Exam 11-08

Signature of the Candidate

I have verified the information filled by the candidate above

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IMPORTANT NOTE:

Wherever a question is being attempted, all its parts/ sub-parts must be attempted contiguous. This means that before moving on to the next

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P.T.O.

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			1
	(b)		—	06
	(c)			09
	(d)			09
	(e)			09
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			06
	(b)			06 12
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5	(a)			08
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6	(a)			08
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	(c)			14
	(d)			
7	(a)			—
	(b)			
	(c)			
	(d)			
8	(a)			—
	(b)			16
	(c)			09
	(d)			14
Total Marks				180/250

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SECTION-A

1. (a) Give an example of an infinite group that has exactly two elements of order 4.

(10)

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- (b) Let $\omega \neq 1$ be a root of $x^3 = 1$. Prove that $T = \{a + bw \in \mathbb{C} \mid a, b \in \mathbb{Q}\}$ is a subfield of the field \mathbb{C} . (10)

Sol. Given $\omega \neq 1$ is a root of $x^3 = 1$
 $\therefore T = \{a + bw \in \mathbb{C} \mid a, b \in \mathbb{Q}\}$.

T is a subfield of \mathbb{C} iff :-

(i) $(T, +)$ forms an abelian subgroup of \mathbb{C} i.e. $a, b \in T \Rightarrow a+b \in T$

(ii) $(T, .)$ forms a semi subgroup of \mathbb{C} i.e. $a, b \in T \Rightarrow ab \in T$.

Check (i) Is $(T, +)$ an abelian subgroup of \mathbb{C} :-

$$\text{Let } p = a + bw, q = c + dw \in T.$$

$$p - q = (a + bw) - (c + dw) = (a - c) + (b - d)w \in T$$

$$\text{Thus, if } a, c \in T \Rightarrow p - q \in T \quad (\because a - c \in \mathbb{Q}, b - d \in \mathbb{Q}).$$

Thus $(T, +)$ is an abelian subgroup of \mathbb{C} . $\text{---} \textcircled{1}$.

Check (ii) Is $(T, .)$ a subsemigroup of \mathbb{C} :-

$$\text{Let } p = a + bw, q = c + dw \in T.$$

$$p \cdot q = (a + bw)(c + dw) = ac + bcw + adw + bdw^2 \quad \text{---} \textcircled{2}$$

Now if ω is a cube root of unity, we have $w + w^2 + 1 = 0$.

$$\Rightarrow w^2 = -1 - w \quad \text{---} \textcircled{3}$$

Thus, from $\textcircled{2}$ & $\textcircled{3}$:-

$$p \cdot q = ac + (bc + ad)w + bd(-1 - w)$$

$$\stackrel{\text{---} \textcircled{4}}{=} (ac - bd) + (bc + ad - bd)w \in T$$

$$\text{Thus, if } p, q \in T \Rightarrow pq \in T \quad (\because ac - bd \in \mathbb{Q}, bc + ad - bd \in \mathbb{Q}).$$

Thus $(T, .)$ is a subsemigroup of \mathbb{C} $\text{---} \textcircled{4}$.

From $\textcircled{1} \& \textcircled{4}$, $(T, +, .)$ is a subfield of \mathbb{C} .

- (c) For what choice of a and b , if any, will the function

$$f(x) = \begin{cases} ax - 6, & \text{if } x > 1 \\ bx^2, & \text{if } x \leq 1 \end{cases}$$

become differentiable at $x = 1$?

(10)

Sol. Given $f(x) = \begin{cases} ax - 6, & x > 1 \\ bx^2, & x \leq 1 \end{cases}$

For $f(x)$ to be differentiable at $x = 1$, $f(x)$ must be continuous at $x = 1$. i.e. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$.

Now $f(1) = b$.

$$\lim_{x \rightarrow 1^+} f(x) = a - 6$$

$$\lim_{x \rightarrow 1^-} f(x) = b.$$

For f to be continuous at $x = 1$,

$$\boxed{b = a - 6}$$

.....①

Thus, we have $f(x) = \begin{cases} ax - 6, & x > 1 \\ (a-6)x^2, & x \leq 1 \end{cases}$

For $f(x)$ to be differentiable at $x = 1$,

LHD of $f(x) = \text{RHD of } f(x)$.

$$\text{LHD of } f(x) = \lim_{x \rightarrow 1^-} 2bx = 2b = 2(a-6)$$

$$\text{RHD of } f(x) = \lim_{x \rightarrow 1^+} a = a.$$

For $f(x)$ to be differentiable at $x = 1$,

$$2(a-6) = a \Rightarrow 2a - 12 = a \Rightarrow \boxed{a = 12} \quad \text{.....②}$$

From ① & ②, $a = 12$, $b = a - 6 = 6$.

Thus $f(x) = \begin{cases} 12x - 6, & x > 1 \\ 6x^2, & x \leq 1 \end{cases}$

∴ The required values of a & b are 12 & 6 respectively.

- (e) Write the dual of the following LPP.

$$\text{Maximize } z = 2x_1 + 5x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + 4x_2 - 3x_3 \leq 8$$

$$-2x_1 - 2x_2 + 3x_3 \geq -7$$

$$x_1 + 3x_2 - 5x_3 \geq -2$$

$$4x_1 + x_2 + 3x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

(10)

(Q5^n) As the constraint is in Maximize form, convert all inequalities to \leq form :-
 As there are four constraints, let them in 4 variables y_1, y_2, y_3, y_4 in the dual.
 $\text{Max. } Z = 2x_1 + 5x_2 + 3x_3$

$$\begin{aligned} \text{st. } & 2x_1 + 4x_2 - 3x_3 \leq 8 & y_1 \\ & 2x_1 + 2x_2 - 3x_3 \leq 7 & y_2 \\ & -x_1 + 3x_2 + 5x_3 \leq 2 & y_3 \\ & 4x_1 + x_2 + 3x_3 \leq 4 & y_4 \\ & x_1, x_2, x_3 \geq 0. & \end{aligned}$$

$$\text{Dual : Min. } Z = 8y_1 + 7y_2 + 2y_3 + 4y_4,$$

$$\begin{aligned} \text{st. } & 2y_1 + 2y_2 - y_3 + 4y_4 \geq 2 \\ & 4y_1 + 2y_2 - 3y_3 + y_4 \geq 5 \\ & -3y_1 - 3y_2 + 5y_3 + 3y_4 \geq 3. \end{aligned} \quad \left. \begin{array}{l} (\text{no equality}) \\ \Leftrightarrow \text{no unshaded variable in primal).} \end{array} \right\}$$

(no unshaded variable \Rightarrow no equality in primal).
 which is the required dual.

2. (a) Suppose G is the group defined by the following Cayley table

(9)

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	8	7	6	5	4	3
3	3	4	5	6	7	8	1	2
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	4	3	2	1	8	7
7	7	8	1	2	3	4	5	6
8	8	7	6	5	4	3	2	1

- (i) Find the centralizer of each member of G .
- (ii) Find $Z(G)$.
- (iii) Find the order of each element of G . How are these orders arithmetically related to the order of the group?

- (b) Consider the element $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in $SL(2, \mathbb{R})$. What is the order of A? If we view $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ as a member of $SL(2, \mathbb{Z}_p)$ (p is a prime), what is the order of A?

Where $SL(2, \mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in GL(2, \mathbb{R}) \mid ad - bc = 1 \right\}$. (8)

(c) Show that $\int_0^{\infty} x^{n-1} e^{-x} dx$ converges iff $n > 0$.

(16)

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(d) Show by the method of contour integration that

$$\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 0. \quad (17)$$



3. (a) Give an example of a field of 9 elements.

(15)

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- (b) Show that the sequence of functions $\{f_n\}$, where $f_n(x) = n \times 1!(1-x)^n$ is not uniformly convergent on $[0, 1]$. **(15)**

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- (c) A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 800 units respectively. Unit transportation costs (in rupees) are given below:

		To				
		D	E	F	G	H
From	A	5	8	6	6	3
	B	4	7	7	6	5
	C	8	4	6	6	4

Determine an optimum distribution for the company in order to minimize the total transportation cost.

(20)

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4. (a) (i) Prove that the group $\frac{4\mathbb{Z}}{12\mathbb{Z}}$ is isomorphic to \mathbb{Z}_3 .
(ii) Show that \mathbb{Z}_9 is not a homomorphic image of \mathbb{Z}_{36} . (12)

$$\text{SOL}^n \quad (i) \quad 4\mathbb{Z} = \{ \dots -8, -4, 0, 4, 8, \dots \}.$$

$$12\mathbb{Z} = \{ -24, -12, 0, 12, 24, \dots \}.$$

$$\frac{4\mathbb{Z}}{12\mathbb{Z}} = \{ 12z + a \mid a \in 4\mathbb{Z} \}.$$

$$12z + 0 = \{ \dots -24, -12, 0, 12, 24, \dots \}$$

$$12z + 4 = \{ \dots, -20, -8, 4, 16, 28, \dots \}$$

$$12z + 8 = \{ \dots, -16, -4, 8, 20, 32, \dots \}$$

$$12z + 12 = \{ \dots, -12, 0, 12, 24, 36, \dots \} = 12z + 0.$$

$$12z + 16 = 12z + 4$$

$$12z + 20 = 12z + 8 \dots$$

$$\text{Thus } \frac{4\mathbb{Z}}{12\mathbb{Z}} = \{ 12z + 0, 12z + 4, 12z + 8 \}.$$

$\frac{4\mathbb{Z}}{12\mathbb{Z}}$	12z + 0	12z + 4	12z + 8
12z + 0	12z + 0	12z + 4	12z + 8
12z + 4	12z + 4	12z + 8	12z + 0
12z + 8	12z + 8	12z + 0	12z + 4

\mathbb{Z}_3	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(Addition defined as $(12z + a) + (12z + b) = 12z + (a+b)$).

→ \mathbb{Z}_3 is a homomorphic image of $\frac{4\mathbb{Z}}{12\mathbb{Z}}$.

- (b) Show that the function f defined on $[0, 1]$ as

$f(x) = 2rx$ if $\frac{1}{r+1} < x < \frac{1}{r}$, $r \in \mathbb{N}$ is integrable over $[0, 1]$ and

$$\int_0^1 f(x) dx = \frac{\pi^2}{6}. \quad (13)$$

Soln. Given that $f(x) = 2rx$ $\frac{1}{r+1} < x < \frac{1}{r}$, $r \in \mathbb{N}$.



At every point $x = \frac{1}{r}$,

$$\text{Lt } f(x) = \frac{2r}{1/r} = \frac{2r}{1/r} = 2(r-1)r.$$

$$\text{Lt } f(x) = 2rx.$$

As $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$, limit does not exist
and thus $f(x)$ is discontinuous at all $x = \frac{1}{\lambda}$.

However, we have a theorem that $f(x)$ is Riemann integrable even if it has infinite points of discontinuity.
Hence, $f(x)$ is Riemann integrable.

For any interval $x \in (\frac{1}{\lambda H}, \frac{1}{\lambda})$

$$\begin{aligned} \int_{\frac{1}{\lambda H}}^{\frac{1}{\lambda}} f(x) dx &= \int_{\frac{1}{\lambda H}}^{\frac{1}{\lambda}} 2x \lambda dx = 2\lambda \left(\frac{x^2}{2}\right) \Big|_{\frac{1}{\lambda H}}^{\frac{1}{\lambda}} \\ &= \lambda \left(\frac{1}{\lambda^2} - \frac{1}{(\lambda H)^2} \right) = \frac{\lambda(2\lambda H + 2H - 1)}{2\lambda(\lambda H)^2} = \boxed{\frac{2\lambda H}{2(\lambda H)^2}} \\ &= \lambda \left(\frac{1}{\lambda^2} - \frac{2H-1}{(\lambda H)^2} \right) = \boxed{\frac{1}{\lambda} - \frac{2}{(\lambda H)^2} + \frac{1}{(\lambda H)^2}} \\ \int_0^1 f(x) dx &= \sum_{n=1}^{\infty} \int_{\frac{1}{\lambda n}}^{\frac{1}{\lambda(n+1)}} f(x) dx = \sum_{n=1}^{\infty} \frac{1}{\lambda} - \sum_{n=1}^{\infty} \frac{1}{\lambda(n+1)} \\ &= \sum_{n=1}^{\infty} 2 \left(\frac{1}{\lambda} + \frac{1}{(\lambda(n+1))^2} \right) + \sum_{n=0}^{\infty} \frac{1}{\lambda(n+1)^2} + \sum_{n=1}^{\infty} \frac{1}{\lambda(n+1)^2} \\ &= \boxed{\frac{\pi^2}{6}} \quad \downarrow \quad \underbrace{2 \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \right)}_{\frac{\pi^2}{12}}. \end{aligned}$$

Hence, shown.

(c) Show that the function defined by

$$f(z) = u + iv = \begin{cases} \frac{\operatorname{Im}(z^2)}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

Satisfies the Cauchy-Riemann equations at the origin, yet it is not differentiable there.

(12)

Sol: The Cauchy Riemann Equations are :-

$$\begin{aligned} u_x &= v_y \\ \text{d} u_y &= -v_x. \end{aligned}$$

$$f(z) = u + iv = \begin{cases} \frac{\operatorname{Im}(z^2)}{z} & , z \neq 0 \\ 0 & , z = 0. \end{cases}$$

$$= \begin{cases} \frac{2ab}{a+ib} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

$$= \begin{cases} \frac{2ab(a+ib)}{a^2+b^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

$$u = \frac{2a^2b}{a^2+b^2}, \quad v = \frac{2ab^2}{a^2+b^2}$$

$$u_x(0,0) = \lim_{h \rightarrow 0} \frac{u(0+h,0) - u(0,0)}{h} = \lim_{h \rightarrow 0} \frac{u(h,0)-0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2(h^2)0}{h^2+b^2}}{h} = 0.$$

$$v_y(0,0) = \lim_{k \rightarrow 0} \frac{v(0,0+k) - v(0,0)}{k} = \lim_{k \rightarrow 0} \frac{v(0,k)-0}{k}$$

$$= \lim_{k \rightarrow 0} \frac{\frac{2(0)k^2}{a^2+b^2}}{k} = 0. \quad \boxed{u_x = v_y} \quad \text{--- (1)}$$

$$v_y(0,0) = \lim_{k \rightarrow 0} \frac{v(0,0+k) - v(0,0)}{k} = \lim_{k \rightarrow 0} \frac{v(0,k)-0}{k}$$

$$= \lim_{k \rightarrow 0} \frac{\frac{2(0)k^2}{a^2+b^2}}{k} = 0.$$

$$V_x(\theta, 0) = \lim_{h \rightarrow 0} \frac{V(0th, 0) - V(0, 0)}{h} = \lim_{h \rightarrow 0} \frac{V(0, h) - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{260^2}{0.2m^2} h}{h} = 0.$$

thus $\boxed{V_y = -V_x}$ —②

from ① & ②, CR equations are satisfied at origin.

To check differentiability, first check continuity at origin.

Approaching origin along $x = a \cos \theta, y = \frac{a \sin \theta}{m}$.

$$f(z) = \lim_{x \rightarrow 0} \frac{2 \cdot a^2 \cos \theta \sin \theta (x e^{x^2})}{x^2} = 0.$$

thus $f(z)$ is continuous.

$$\left. \frac{df(z)}{dz} \right|_{z=0} = \lim_{(a+b) \rightarrow 0} \frac{f(a+b) - f(0, 0)}{a+b} = \lim_{(a+b) \rightarrow 0} \frac{2ab}{a^2 + b^2}.$$

Approaching along $b = ma$:- $f'(z) = \frac{2a \cdot ma}{a^2 + m^2 a^2} = \frac{2m}{1+m^2} = f(m)$

Thus $f(z)$ is not differentiable as $f'(z)$ gives different values

- (d) Let $x_1 = 2, x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations

along different directions.

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18.$$

Reduce the given feasible solution to a basic feasible solution. (13)

Soln let us first write the equations in vector form:-

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} -1 \\ 4 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 0 \end{bmatrix} x_3 = \begin{bmatrix} 2 \\ 18 \end{bmatrix}.$$

A1 $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ & $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ are LD

$$\Rightarrow \lambda_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} -1 \\ 4 \end{bmatrix} + \lambda_3 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0 \quad \cancel{\lambda_1, \lambda_2, \lambda_3 \neq 0}$$

let $\lambda_3 = 2$:-

$$\lambda_1 = -\frac{16}{3}, \quad \lambda_2 = \frac{4}{3}.$$

12 $\min \left(\frac{x_1}{\lambda_1}, \frac{x_2}{\lambda_2}, \frac{x_3}{\lambda_3} \right) = \min \left(\frac{2}{-16/3}, \frac{4}{4/3}, \frac{1}{2} \right) = \boxed{\frac{-9}{8}}$

$$x_1 - \alpha \lambda_1 = 2 - \left(\frac{-3}{2}\right) \left(-\frac{16}{3}\right) = 0$$

$$x_2 - \alpha \lambda_2 = 4 - \left(\frac{-3}{2}\right) \left(\frac{4}{3}\right) = \frac{9}{2}$$

$$x_3 - \alpha \lambda_3 = 1 - \left(\frac{-3}{2}\right) (2) = \frac{13}{2}.$$

∴ $\left(0, \frac{9}{2}, \frac{13}{2}\right) \in (x_1, x_2, x_3)$ is the required
basic feasible solution of the given system

SECTION-B

5. (a) Let G be a group and $a, b \in G$, such that $ab = ba$ and $O(a)$ and $O(b)$ are relatively Prime. Then Prove that $O(ab) = O(a)O(b)$. (10)

Sol: Given that $ab = ba \Rightarrow (ab)^p = a^p b^p$ for any p .

$$\text{Let } O(a) = m, \quad O(b) = n, \quad O(ab) = \alpha.$$

$$(1) \quad (ab)^{mn} = a^{mn} b^{mn} = (a^m)^n (b^n)^m = e^n e^m = e.$$

$$\Rightarrow O(ab) | mn \quad \text{---} \textcircled{1}$$

$$(2) \quad \text{Given that } (m, n) = 1$$

$$\Rightarrow \exists x, y \in \mathbb{Z} \text{ s.t. } mx + ny = 1.$$

$$a^{mx+ny} = a \Rightarrow a = a^{mx} \cdot a^{ny} = a^{ny}$$

$$b^{mx+ny} = b \Rightarrow b = b^{mn} \cdot b^{ny} = b^{ny}$$

$$ab = a^{ny} b^{ny} \Rightarrow (ab)^p = e = a^{npy} b^{npy}$$

$$\Rightarrow m | npy \Rightarrow m | p \quad (\because (m, n) = 1).$$

$$\text{and } n | npy \Rightarrow n | p \quad (\because (m, n) = 1)$$

$$\Rightarrow mn | p \Rightarrow mn | O(a, b) \quad \text{---} \textcircled{2}$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2}: \quad O(ab) = mn$$

$$\Rightarrow \boxed{O(ab) = O(a)O(b)}$$

Kenya, proof.

(b) Show that $\mathbb{Z}[\sqrt{-7}]$ is not a U.F.D.

(10)

Soln " $\mathbb{Z}[\sqrt{-7}] = \mathbb{Z}[\tau_i] = \underbrace{\{a+b\tau_i \mid a, b \in \mathbb{Z}\}}$

Any element of $\mathbb{Z}[\sqrt{-7}]$ is really $\Rightarrow \boxed{a = \pm 1}$
 $\boxed{b = 0}$.

Let $(a+\tau b) = (c+\tau d)(e+\tau f)$.

$\underline{(a-\tau b)} = \underline{(c-\tau d)(e-\tau f)}$,

$\Rightarrow a^2 - 4ab^2 = (c^2 - 4cd^2)(e^2 - 4ef^2)$.

$\Rightarrow a = \pm 1, b = 0$

Or $\mathbb{Z}[\sqrt{-7}]$ is irreducible.

But $\tau(\sqrt{7})$ is not prime.

Hence $\tau(\sqrt{7})$ is UFD.

Please
practise it
again

(c) Examine the convergence of the integral

$$\int_2^3 \frac{dx}{(x-2)^{\frac{1}{4}}(3-x)^2} \quad (10)$$

Soln. To check the convergence of the integral

$$\int_2^3 \frac{dx}{(x-2)^{\frac{1}{4}}(3-x)^2}$$

(Let us make the substitution $x-2 = y$)

$$x-2 = y \Rightarrow dx = dy$$

$$3-x = 1-y$$

$$\Rightarrow \int_2^3 \frac{dx}{(x-2)^{\frac{1}{4}}(3-x)^2} = \int_0^1 \frac{dy}{y^{\frac{1}{4}}(1-y)^2} = \int_0^1 y^{-\frac{1}{4}}(1-y)^{-2} dy$$

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$$\text{Now, } B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx.$$

$$\text{Thus } \int_2^3 \frac{dx}{(x-2)^{\frac{1}{2}} (3-x)^2} = B\left(\frac{3}{2}, -1\right).$$

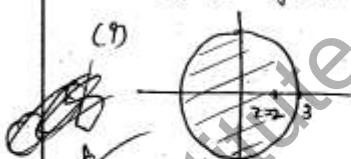
$B\left(\frac{3}{2}, -1\right)$ is not convergent

$$\text{Hence } \int_2^3 \frac{dx}{(x-2)^{\frac{1}{2}} (3-x)^2} \text{ is not convergent.}$$

(d) If $f(z) = \frac{z^2 + 5z + 6}{z-2}$, does Cauchy's theorem apply. (10)

- (i) When the path of integration C is a circle of radius 3 with origin as centre.
- (ii) When C is a circle of radius 1 with origin as centre.

Sol: For Cauchy's theorem to apply, the pole (or singularity) of the integrand must lie outside the path of integration. In the given case, $z=2$ is the pole.



$$z=2 \in (|z| < 3).$$

Hence Cauchy theorem can be applied.



$$z=2 \notin (|z| < 1).$$

Hence Cauchy theorem cannot be applied.

In case (i), $\int_{|z|=3} f(z) dz = 2\pi i f(z) = 2\pi i (z^2 + 10 + 6)$
 $= 2\pi i (z_0) = \boxed{40\pi i}$

In case (ii) $\int_{|z|=1} \frac{z^2 + 5z + 6}{z-2} dz$ is analytic inside the contour

Hence, Cauchy's theorem does not apply and

$$\int_C f(z) dz = \boxed{0}$$

Thus, Cauchy's theorem applies in (i) but not in (ii).

- (c) Show that the following system of linear equations has degenerate solution:

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3.$$

(10)

Sol. Let us find all possible basic solutions and then check which are degenerate :-

Choose $x_3 = 0$

$$2x_1 + x_2 = 2$$

$$3x_1 + 2x_2 = 3$$

which has the solution $(x_1, x_2) = (1, 0)$.

Thus $(x_1, x_2, x_3) = (1, 0, 0)$

→ ①

Degenerate
Basic Solution

Choose $x_2 = 0$

$$2x_1 - x_3 = 2$$

$$3x_1 + x_3 = 3.$$

which has the solution $(x_1, x_3) = (1, 0)$

Thus $(x_1, x_2, x_3) = \underbrace{(1, 0, 0)}_{\text{D}} \quad \begin{cases} \text{Degenerate} \\ \text{Basic Solution} \end{cases}$

Q9 -

choose $x_4 = 0$

$$x_2 - x_3 = 2$$

$$2x_2 + x_3 = 3$$

which has the solution $(x_2, x_3) = \left(\frac{5}{3}, -\frac{1}{3}\right)$

$$\text{Thus } (x_1, x_2, x_3) = \left(0, \frac{5}{3}, -\frac{1}{3}\right)$$

Non-degenerate
Basic solution

From ①, ② & ③, the system has 2 dependent
& 1 non-degenerate basic solution.

6. (a) In the group S_3 , show that the subset
 $H = \{a \in S_3 / o(a) \text{ divides } 2\}$ is not a subgroup.

(10)

Sol. Given the group $S_3 = \{(1), (12), (23), (13), (123), (132)\}$.

$$o(a) | 2 \Rightarrow o(a) = 1 \text{ or } o(a) = 2.$$

(i) $o((1)) = 1 \in S_3.$

(ii) $(12)(12) = (1) \Rightarrow o((12)) = 2 \notin S_3.$

(iii) $(23)(23) = (1) \Rightarrow o((23)) = 2 \notin S_3.$

(iv) $(13)(13) = (1) \Rightarrow o((13)) = 2 \notin S_3.$

(v) $(123)(123) = (132)$
 $(123)(132) = (1) \quad] \quad o((123)) = 3 \notin S_3$

(vi) $(132)(132) = (123)$
 $(132)(123) = (1) \quad] \quad o((123)) = 3 \notin S_3.$

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$$H = \{(1), (12), (13), (23)\}.$$

Check for closure $(12)(13) = (132) \notin H.$

Thus H is not closed under ~~permutation~~ multiplication

Hence $H \neq S_3$. Hence, proved,

- (b) Let T denote the group of all non-singular upper triangular 2×2 matrices with real entries i.e., the matrices of the

form, $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ where $a, b, c \in \mathbb{R}$ and $ac \neq 0$. Show that $H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \mid x \in \mathbb{R} \right\}$ is a normal subgroup of T .

(10)

Soln. Given mat $T = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$.

$$H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \mid x \in \mathbb{R} \right\}$$

If $h \in H$ then $\forall x \in T, h \in H$

$$xhx^{-1} \in H.$$

Let $h = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \in H.$

$$x = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \in T.$$

Now, $x^{-1} = \frac{1}{ac} \begin{bmatrix} c & -b \\ 0 & a \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{ac} \\ 0 & \frac{1}{c} \end{bmatrix}.$

$$\begin{aligned}
 \text{Now, } x h_1 x^{-1} &= \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/a & -b/a \\ 0 & 1/c \end{bmatrix} \\
 &= \begin{bmatrix} a & ab+b \\ 0 & c \end{bmatrix} \begin{bmatrix} 1/a & -b/a \\ 0 & 1/c \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -\frac{b}{c} + \frac{ab+b}{c} \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & \cancel{\frac{ab}{c}} \\ 0 & 1 \end{bmatrix} \in H. \quad \text{---(1)}
 \end{aligned}$$

(P) Now check that $H \neq \text{high}_2 \in H$, $x h_2 x^{-1} \in H$.

$$h_1 = \begin{bmatrix} 1 & h_1 \\ 0 & 1 \end{bmatrix}, \quad h_2 = \begin{bmatrix} 0 & h_2 \\ 0 & 1 \end{bmatrix}, \quad h_1 h_2 = \begin{bmatrix} 1 & -h_2 \\ 0 & 1 \end{bmatrix}$$

$$h_1 h_2^{-1} = \begin{bmatrix} 1 & h_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -h_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & h_1 - h_2 \\ 0 & 1 \end{bmatrix} \in H. \quad \text{---(2)}$$

From (2), $h_1, h_2 \in H \Rightarrow h_1 h_2^{-1} \in H$. Thus $H \trianglelefteq T$.

From (1), $x \notin T, x h_1 x^{-1} \notin x h_2 x^{-1} \in H$. Thus $H \triangleleft T$.

Hence it has been shown that H is a normal subgroup of T . \checkmark

- (c) Let \mathbf{R} be the set of all real numbers and \mathbf{R} be the set of all real-valued continuous functions defined on \mathbf{R} . Define $(f+g)(x) = f(x) + g(x)$ and $(fg)(x) = f(x)g(x)$ for all $f, g \in \mathbf{R}$ and for all $x \in \mathbf{R}$. Show that $(\mathbf{R}, +, \cdot)$ is a ring under the binary operations defined above.

(15)

Sol: Then, $\mathbf{R} = \{ f(x) \mid f: \mathbf{R} \rightarrow \mathbf{R} \text{ and } f \text{ is continuous} \}$.

(i) First show that b.o. are well-defined :-

$$\text{let } f' = f, g' = g.$$

$$\textcircled{a} (f' + g')(x) = f'(x) + g'(x) = f(x) + g(x) = (f+g)(x)$$

Thus addition is well defined.

$$\textcircled{b} f'g'(x) = f'(x)g'(x) = f(x)g(x) = fg(x)$$

Thus multiplication is well defined.

Now, we show that $(\mathbf{R}, +)$ is an abelian group :-

$$\textcircled{a} \text{ closure } (f+g)(x) = f(x) + g(x).$$

As sum of continuous functions is again continuous, $f+g \in \mathbf{R}$.

$$\textcircled{b} \text{ associativity } (f+(g+h))(x) = f(x) + (g+h)(x)$$

$$= f(x) + g(x) + h(x)$$

$$= (f(x) + g(x)) + h(x)$$

$$= (f+g)+h(x),$$

thus \mathbf{R} is associative over $+$.

$$\textcircled{c} \text{ Identity } \exists 0(x) = 0 \text{ s.t. } (f+0)(x) = (0+f)(x)$$

thus \mathbf{R} has $0(x) = 0$ as identity.

$$\textcircled{d} \text{ inverse } \text{for every } f \text{ fr }$$

$$\exists -f \in \mathbf{R} \text{ s.t. } (f+(-f))(x) = (-f+f)(x)$$

$$= 0(x) = I.$$

thus \mathbf{R} has inverse for every element under $+$.

$$\textcircled{e} \text{ Commutativity } (f+g)(x) = f(x) + g(x) = g(x) + f(x) = (g+f)(x)$$

thus \mathbf{R} is commutative over addition.

Thus, $(R, +)$ is an abelian group — ①

(iii). Now let us show that (R, \cdot) is a semigroup -

② Closure $fg(x) = f(x)g(x)$

Product of continuous functions is continuous
Hence $fg \in R$ Thus fg is closed.

③ Associativity $f(g \cdot h)(x) = f(x)gh(x) = f(x)g(x)h(x)$
 $= fg(x)h(x) = (fg)h(x).$

Thus R is associative over \cdot .

Thus, (R, \cdot) is a semigroup — ②

(iv) Now, let us show . distributes over $+$:-

④ $f \cdot (g+h)(x) = f(x)(g+h)(x) = f(x) \cdot (g(x)+h(x))$
 $= f(x) \cdot g(x) + f(x) \cdot h(x)$

⑤ $(f+g) \cdot h(x) = (f+g)(x) \cdot h(x) = (f(x)+g(x)) \cdot h(x)$
 $= f(x) \cdot h(x) + g(x) \cdot h(x)$

thus . distributes over $+$ in R — ③

From ①, ②, ③:-

(i) $(R, +)$ is an abelian group

(ii) (R, \cdot) is a semigroup

(iii) In R , . distributes over $+$.

Hence proved that $(R, +, \cdot)$ is a very under the given binary operations.

(d) Show that $\mathbb{Z}[i] = \left\{ m + ni/m, n \in \mathbb{Z}, i = \sqrt{-1} \right\}$ is a Euclidean domain.

(15)

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7. (a) Is the intersection of an arbitrary collection of open sets open? Justify your answer by a proof or by a counter example. (12)

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(b) Discuss the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \dots$$

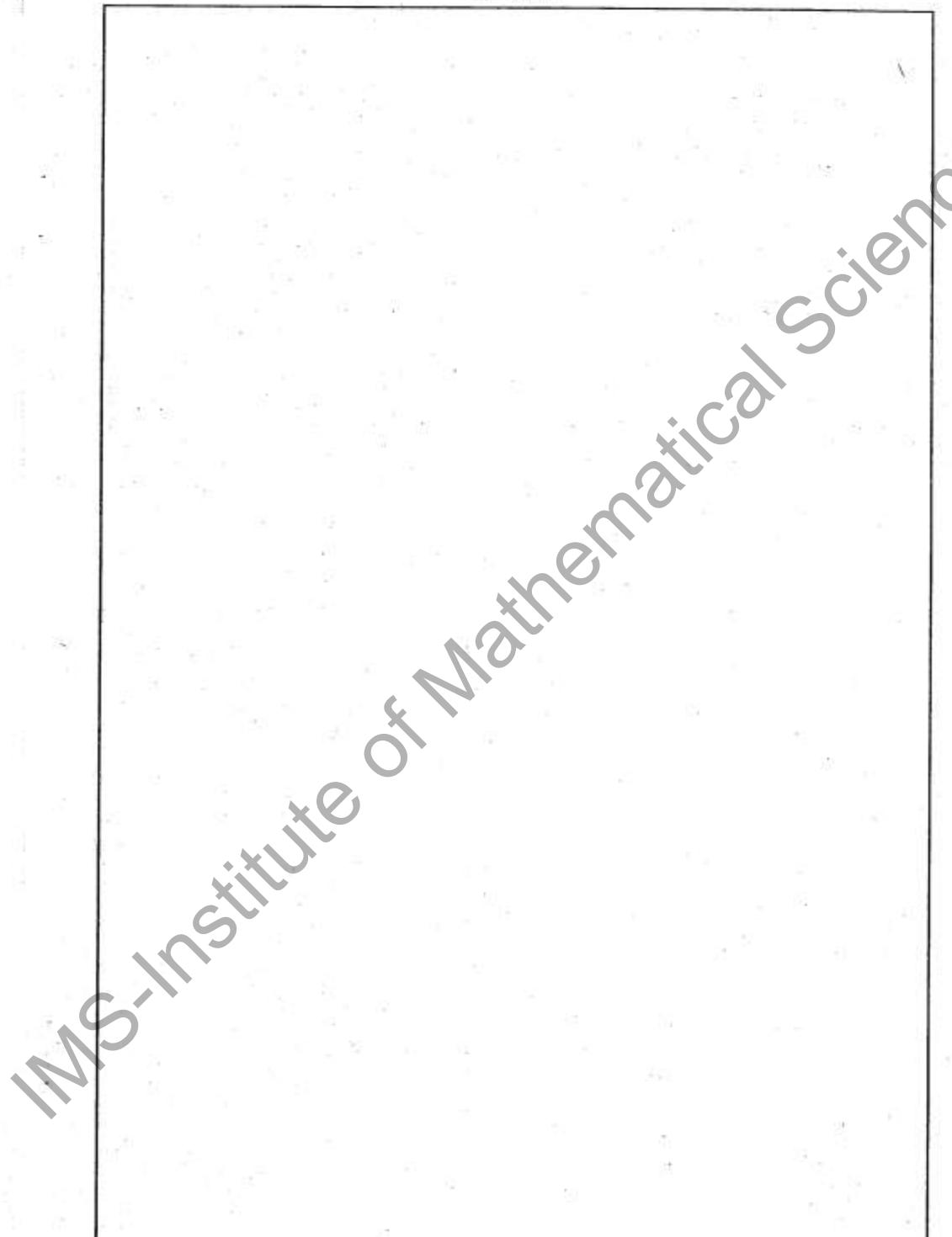
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(c) Find the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

(8)

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(d) Examine for uniform convergence and continuity of the limit function of the sequence $\langle f_n \rangle$, where

$$f_n(x) = \frac{nx}{1+n^2x^2}, 0 \leq x \leq 1. \quad (15)$$

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8. (a) If $a > c$, use Rouche's theorem to prove that the equation $e^z = az^n$ has n roots inside the circle $|z| = 1$.

(10)

- (b) Find the Taylor's or Laurent's series which represent the function $\frac{1}{(1+z^2)(z+2)}$. (15)

- (i) when $|z| < 1$,
- (ii) when $1 < |z| < 2$,
- (iii) when $|z| > 2$

Sol? we have to find the Taylor's (or Laurent's) series that represents

$$\frac{1}{(1+z^2)(z+2)} = \frac{1}{(z+1)(z-1)(z+2)} = \frac{\frac{1}{5} \frac{1}{(z+1)} - \frac{1}{5} \frac{3-2}{z^2+1}}$$

- (i) when $|z| < 1$.

$$\Rightarrow \frac{|z|}{2} < 1 \text{ and } |z|^2 < 1.$$

$$\begin{aligned} \frac{1}{(1+z^2)(z+1)} &= \frac{1}{5} \left(\frac{1}{z+1} - \frac{1}{5} \frac{3-2}{z^2+1} \right) \\ &= \frac{1}{5} \frac{1}{2} \frac{1}{\left(1+\frac{z}{2}\right)} - \frac{1}{5} \frac{(z-1)}{\left(1+\frac{z}{2}\right)} \\ &= \frac{1}{10} \left(1 + \frac{z}{2}\right)^{-1} - \frac{1}{5} \frac{(z-1)}{\left(1 + \frac{z}{2}\right)^{-1}} \\ &= \frac{1}{10} \left(1 - \frac{3}{2} + \frac{3}{4} - \frac{3^3}{8} + \dots\right) - \frac{1}{5} (z-1) \left(1 - z^2 + z^4 - z^6 + \dots\right) \\ &= \frac{1}{10} \left(1 - \frac{3}{2} + \frac{3}{4} - \frac{3^3}{8} + \dots\right) - \frac{1}{5} (z-1) (2 + z + 2z^2 - z^3 - 2z^4 + z^5 + \dots) \\ &= \left(\frac{1}{10} + \frac{z}{5}\right) + z \left(-\frac{1}{10} - \frac{1}{5}\right) + z^2 \left(\frac{1}{10} - \frac{2}{5}\right) + z^3 \left(-\frac{1}{10} + \frac{1}{5}\right) + \dots \end{aligned}$$

$$= \boxed{\frac{1}{2} - \frac{3}{4} z - \frac{3}{2} z^2 + \frac{3}{16} z^3 + \dots}$$

- (ii) when $1 < |z| < 2$

$$\Rightarrow \frac{1}{|z|} < 1 \Rightarrow \frac{1}{|z|^2} < 1 \text{ and } \frac{1}{|z|} < 1$$

$$\begin{aligned}
 \frac{1}{(1+3^2)(3+2)} &= \frac{1}{5} \left(\frac{1}{3+2} \right) - \frac{1}{5} \frac{3^{-2}}{3^2 n} \\
 &= \frac{1}{5} \cdot \frac{1}{2} \left(\frac{1}{1+\frac{2}{3}} \right) - \frac{1}{5} \frac{3^{-2}}{3^2 \left(1+\frac{1}{3^2} \right)} \\
 &= \frac{1}{10} \left(1+\frac{2}{3} \right)^{-1} - \frac{1}{5 \cdot 3^2} \left(1+\frac{1}{3^2} \right)^{-1} = \frac{1}{10} \left(1 - \frac{3}{2} + \frac{3^2 - 3^3}{8} + \dots \right) \\
 &\quad + -\frac{1}{5 \cdot 3^2} \left(1 - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{3^4} + \dots \right) \\
 &= \frac{1}{10} \left(1 - \frac{3}{2} + \frac{3^2 - 3^3}{8} \right) - \frac{1}{5 \cdot 3^2} \left[\left(3 - \frac{1}{3} + \frac{1}{3^3} - \frac{1}{3^5} + \dots \right) + \left(-2 + \frac{2}{3^2} - \frac{2}{3^4} + \frac{2}{3^6} - \dots \right) \right] \\
 &= \frac{1}{10} \left(1 - \frac{3}{2} + \frac{3^2 - 3^3}{8} \right) - \frac{1}{5} \left[\left(\frac{1}{3} - \frac{1}{3^3} + \frac{1}{3^5} - \frac{1}{3^7} + \dots \right) + \left(-\frac{2}{3^2} + \frac{2}{3^4} - \frac{2}{3^6} + \frac{2}{3^8} - \dots \right) \right] \\
 &= \frac{1}{10} \left(\frac{-1}{3} + \frac{2}{3^2} + \frac{1}{5 \cdot 3^3} - \frac{1}{5 \cdot 3^5} + \frac{1}{5 \cdot 3^7} + \dots \right) \\
 &\quad + \frac{1}{10} \left(\frac{1}{3} - \frac{1}{3^3} + \frac{1}{3^5} - \frac{1}{3^7} + \dots \right) + \left(-\frac{2}{3^2} + \frac{2}{3^4} - \frac{2}{3^6} + \frac{2}{3^8} - \dots \right).
 \end{aligned}$$

(iii) $|z_1| > 2$

$$\Rightarrow \left| \frac{2}{3} \right| < 1 \text{ and } \left| \frac{4}{3^2} \right| < 1 \Rightarrow \left| \frac{1}{3^2} \right| < 1.$$

$$\frac{1}{(1+3^2)(3+2)} = \frac{1}{5} \left(\frac{1}{3+2} \right) - \frac{1}{5} \frac{(3-1)}{(3+1)}.$$

$$= \frac{1}{5} \frac{1}{3} \left(\frac{1}{1+\frac{2}{3}} \right) + \left(-\frac{1}{5 \cdot 3} + \frac{2}{5 \cdot 3^2} + \frac{1}{5 \cdot 3^3} - \frac{2}{5 \cdot 3^4} + \dots \right)$$

$$= \frac{1}{5 \cdot 3} \left(1 - \frac{2}{3} + \frac{4}{3^2} - \frac{8}{3^3} + \frac{16}{3^4} - \dots \right) + \left(-\frac{1}{5 \cdot 3} \frac{2}{3^2} + \frac{1}{5 \cdot 3^3} - \frac{2}{5 \cdot 3^4} + \dots \right).$$

$$= \left(\frac{1}{5 \cdot 3} - \frac{2}{5 \cdot 3^2} + \frac{4}{5 \cdot 3^3} - \frac{8}{5 \cdot 3^4} + \frac{16}{5 \cdot 3^5} - \dots \right) + \left(-\frac{1}{5 \cdot 3} \frac{2}{3^2} + \frac{1}{5 \cdot 3^3} - \frac{2}{5 \cdot 3^4} + \dots \right)$$

$$= \boxed{\frac{1}{3^3} - \frac{2}{3^4} + \frac{3}{3^5} + \dots}$$

(c) Solve the following assignment problem

(10)

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

Soln: Given

	A	B	C D.
I	1	4	6 3
II	9	7	10 9
III	4	5	11 7
IV	8	7	8 5.

(i) Subtract all row elements from the first row element

	A	B	C D
I	0	3	5 2
II	2	0	3 2
III	0	1	7 3
IV	3	2	3 0

(ii) subtract all elements of column C from the lowest element

	A	B	C D
I	0	3	2 2
II	-4	0	0 2
III	0	1	4 3
IV	3	-2	0 0 ..

(iii) All the zeroes can be gotten by 3 < 4 zeros \Rightarrow subtract 1 from all remaining elements & add 1 to the vector

	A	B	C D
I	0	2	1 1
II	3	0	0 2
III	0	0	3 2
IV	4	2	0 0 ..

All zeroes can be gotten
by a minimum of 4
rows.

(v) In column row I, only one zero \Rightarrow assign A to I.
 \Rightarrow II-A cannot be assigned.
 Now in row III, only one zero remains \Rightarrow assign B to III \Rightarrow II-B can not be assigned
 Now for row II, only one zero remains \Rightarrow assign C to II \Rightarrow II-C cannot be assigned
 only one zero remains \Rightarrow assign D to IV

Assignment matrix

I - A
 II - C
 III - B
 IV - D

$$\text{Minimum Cost} = 1 + 5 + 10 + 5 \\ = \boxed{21}$$

- (d) Solve the following LP problem by simplex method.

(15)

$$\text{Minimise } z = 8x_1 - 2x_2$$

Subject to

$$-4x_1 + 2x_2 \leq 1$$

$$5x_1 - 4x_2 \leq 3$$

$$x_1, x_2 \geq 0.$$

Sol: (P) First convert the given problem to maxmize type.

$$\text{Max. } z = -8x_1 + 2x_2$$

(ii) Add slack variables to the two equations :-

$$-4x_1 + 2x_2 + 1s_1 + 0s_2 = 1$$

$$5x_1 - 4x_2 + 0s_1 + 1s_2 = 3$$

$$x_1, x_2 \geq 0.$$

$$\text{Max } z = -8x_1 + 2x_2 + 0s_1 + 0s_2$$

The system is now in standard form
 and can be transferred to the simplex table:-

C_i	-8	2	0	0	b	0
x_1	-4	2	1	0	1	$\frac{1}{2}$
x_2	5	-4	0	1	3	-3/4
Z_j	0	0	0	0		
$C_i - Z_j$	-8	2	0	0		
---	---	---	---	---	---	---
x_2	-2	1	$\frac{1}{2}$	0	$\frac{1}{2}$	
x_2	-3	0	2	1	5	
Z_j	-4	2	1	0		1.
$C_i - Z_j$	-4	0	-1	0		

All $C_i \leq 0$. Hence solution is optimal

Thus $x_2 = \frac{1}{2}, s_2 = 5$.

$$5x_1 - 4x_2 + s_1 = 3.$$

$$\Rightarrow 5x_1 - 2 + 5 = 3 \Rightarrow \boxed{x_1 = 0}.$$

$$(x_1, x_2, s_1, s_2) = (0, \frac{1}{2}, 0, 5).$$

Thus to minimize $g = 8x_1 - 2x_2$ with the given constraints, the value of $x_1 = 0$ & the value of $x_2 = \frac{1}{2}$, the minimum value of $g = -1$.

ROUGH SPACE

ROUGH SPACE

$$\textcircled{5} \quad \textcircled{5} \quad \begin{matrix} 20 \\ 5/20 \\ 10/20 \end{matrix} \quad (ab)^{0(ab)} = e$$

$$(ab)^{\frac{m+n}{m+n}}$$

$$mn+ny = 1.$$

$$((ab)^{mn})^{mn} = ab.$$

$$(a^m b^m) (a^m b^m)$$

$$a^m + b^m + 1(ab) = ab$$

$$(ab)^{mn} =$$

$$= ab^{(mn+m+n)} = a^{mn} \cdot a^{m+n} \cdot b^{mn} \cdot b^{m+n} = e.$$

$$\frac{2x+2-1}{x(x+1)^2}$$

$$- \frac{1}{x(x+1)^2} \quad a^{mn} \cdot b^{mn}$$

$$\boxed{mn = n(ab)}$$

$$\frac{2(2+1)}{x(x+1)^2} - \frac{1}{x(x+1)^2}$$

$$2\left(\frac{1}{x} - \frac{1}{x^2}\right)$$

$$\boxed{0(ab) + mn}.$$

$$x^3 + 3y^3 + 3xy(x+y),$$

$$x^3 - iy^3$$

$$(-i)(-i) = \textcircled{1}$$

$$6xy - x^2 = 3x^2 + 3y^2 - 3x^2$$

$$yx = 3x^2 - 3x^2$$

$$(x+iy), (ony)^3$$

$$2un = 12xy$$

$$\boxed{Un = 6xy}$$

$$i^3 x^3 - y^3 - 3xy(i^2 - y)$$

$$(6+8i)^3 = 4^3 + i^3 x^3.$$

$$3y^2 - 3x^2$$

$$- 3y^2 (8i)^2$$

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+ 3xy^2 + 3xy^2

$$- 3y^2$$

P.T.O.

$$a^{mn+pq} = a^{m+n} a^{p+q} \\ = a^{m+p} = a.$$

$$\frac{3^{\alpha} - (5^{-\alpha})}{5}$$

$$(a^b)^c = (a^{bx} b^c)^{y^z}$$

$$c = a^{m_1} b^{n_1}$$

$$-\frac{1}{z} \left(\frac{z-2}{1+z} - \frac{1}{z+2} \right).$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$(ab)^m = \underbrace{ab \cdots ab}_{m \text{ terms}}.$$

$$\frac{2\pi a - 1 - \lambda}{1 + x^2}$$

$$\frac{1}{T} \left(\frac{1}{3+2} \right) - \frac{1}{T} \frac{\frac{3-4}{1+2}}{3+2} = y. \quad (3 - 4y + 2).$$

$$\frac{2x+1}{x(x+1)^2} = \int_0^{\infty} y^{44} (1-y)^{-1} (a,b) = 1.$$

$$\int_0^{\frac{a}{b}} y^{1/a} (1-y)^{-2} dy \quad ab = 1$$

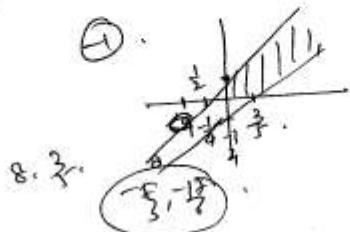
$$(ab)^{ax+by} = ab.$$

10月 23

$$\textcircled{b} \quad a^m \cdot a^n = a^{m+n}$$

$$a^{by} \cdot b^{ax} = a^b \cdot$$

$$a^{by-1} \cdot b^{ax-1} =$$



$$\frac{1}{2}(H)^{-1} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & -b/a \\ b/a & 1 \end{pmatrix} = \begin{pmatrix} c & 0 \\ -b & a \end{pmatrix}$$

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