

Previous Years Questions (1983–2011)

Segment-wise

Ordinary Differential Equations and Laplace Transforms

(According to the New Syllabus Pattern) Paper - I

1983

- ✓ Solve $x \frac{d^2 y}{dx^2} + (x-1) \frac{dy}{dx} - y = x^2$.
- ✓ Solve $(y^2 + yz) dx + (xz + z^2) dy + (y^2 - xy) dz = 0$.
- ✓ Solve the equation $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = t$ by the method of Laplace transform, given that $y = -3$ when $t = 0$, $y = -1$ when $t = 1$.

1984

- ✓ Solve $\frac{d^2 y}{dx^2} + y = \sec x$.
- ✓ Using the transformation $y = \frac{u}{x^k}$, solve the equation $x y'' + (1+2k) y' + xy = 0$.
- ✓ Solve the equation $(D^2 + 1)x = t \cos 2t$, given that $x_0 = x_1 = 0$ by the method of Laplace transform.

1985

- ✓ Consider the equation $y' + 5y = 2$. Find that solution f of the equation which satisfies $f(1) = 3f'(0)$.
- ✓ Use Laplace transform to solve the differential equation $x'' - 2x' + x = e^t$, $\left(' = \frac{d}{dt} \right)$ such that $x(0) = 2, x'(0) = -1$.
- ✓ For two functions f, g both absolutely integrable on $(-\infty, \infty)$, define the convolution $f * g$. If $L(f), L(g)$ are the Laplace transforms of f, g show that $L(f * g) = L(f) \cdot L(g)$.
- ✓ Find the Laplace transform of the function $f(t) = \begin{cases} 1 & 2np \leq t < (2n+1)p \\ -1 & (2n+1)p \leq t \leq (2n+2)p \end{cases}$
 $n = 0, 1, 2, \dots$

1987

- ✓ Solve the equation $x \frac{d^2 y}{dx^2} + (1-x) \frac{dy}{dx} = y + e^x$
- ✓ If $f(t) = t^{p-1}$, $g(t) = t^{q-1}$ for $t > 0$ but $f(t) = g(t) = 0$ for $t \leq 0$, and $h(t) = f * g$, the convolution of f, g show that $h(t) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} t^{p+q-1}; t \geq 0$ and p, q are positive constants. Hence deduce the formula $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

1988

- ✓ Solve the differential equation $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = 2e^x \sin x$.
- ✓ Show that the equation $(12x+7y+1) dx + (7x+4y+1) dy = 0$ represents a family of curves having as asymptotes the lines $3x+2y-1=0, 2x+y+1=0$.
- ✓ Obtain the differential equation of all circles in a plane in the form $\frac{d^3 y}{dx^3} \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\} - 3 \frac{dy}{dx} \left(\frac{d^2 y}{dx^2} \right)^2 = 0$.

1989

- ✓ Find the value of y which satisfies the equation $(xy^2 - y^3 - x^2 e^x) + 3xy^2 \frac{dy}{dx} = 0$; given that $y=1$ when $x=1$.
- ✓ Prove that the differential equation of all parabolas lying in a plane is $\frac{d}{dx} \left(\frac{d^2 y}{dx^2} \right)^{-\frac{2}{3}} = 0$.
- ✓ Solve the differential equation $\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} = 1 + x^2$.

1990

- ✓ (a) If the equation $\lambda^n + a_1 \lambda^{n-1} + \dots + a_n = 0$ (in unknown λ) has distinct roots $\lambda_1, \lambda_2, \dots, \lambda_n$. Show that the constant coefficients of differential equation

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} \frac{dy}{dx} + a_n = b \text{ has the}$$

most general solution of the form

$$y = c_0(x) + c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} + \dots + c_n e^{\lambda_n x}.$$

where c_1, c_2, \dots, c_n are parameters. what is $c_0(x)$?

- ✓ (b) Analyse the situation where the λ – equation in (a) has repeated roots.
✓ Solve the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + y = 0 \text{ is explicit form. If your}$$

answer contains imaginary quantities, recast it in a form free of those.

- ✓ Show that if the function $\frac{1}{t - f(t)}$ can be integrated

(w.r.t 't'), then one can solve $\frac{dy}{dx} = f(\frac{y}{x})$, for any given f . Hence or otherwise.

$$\frac{dy}{dx} + \frac{x-3y+2}{3x-y+6} = 0$$

- ✓ Verify that $y = (\sin^{-1} x)^2$ is a solution of $(1-x^2) \frac{d^2 y}{dx^2}$

$$-x \frac{dy}{dx} = 2. \text{ Find also the most general solution.}$$

1991

- ✓ If the equation $Mdx + Ndy = 0$ is of the form $f_1(xy)$.

$ydx + f_2(xy) \cdot x dy = 0$, then show that $\frac{1}{Mx - Ny}$ is an integrating factor provided $Mx - Ny \neq 0$.

- ✓ Solve the differential equation.

$$(x^2 - 2x + 2y^2) dx + 2xy dy = 0.$$

- ✓ Given that the differential equation $(2x^2 y^2 + y) dx - (x^3 y - 3x) dy = 0$ has an integrating factor of the form $x^k y^k$, find its general solution.

- ✓ Solve $\frac{d^4 y}{dx^4} - m^4 y = \sin mx$.

- ✓ Solve the differential equation

$$\frac{d^4 y}{dx^4} - 2 \frac{d^3 y}{dx^3} + 5 \frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 4y = e^x.$$

- ✓ Solve the differential equation

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} - 5y = xe^{-x}, \text{ given that } y = 0 \text{ and } \frac{dy}{dx} = 0, \text{ when } x = 0.$$

1992

- ✓ By eliminating the constants a, b obtain the differential equation of which $xy = ae^x + be^{-x} + x^2$ is a solution.

- ✓ Find the orthogonal trajectories of the family of semicubical parabolas $ay^2 = x^3$, where a is a variable parameter.

- ✓ Show that $(4x+3y+1) dx + (3x+2y+1) dy = 0$ represents hyperbolas having the following lines as asymptotes

$$x+y=0, 2x+y+1=0. \quad (1998)$$

- ✓ Solve the following differential equation $y(1+xy) dx + x(1-xy) dy = 0$.

- ✓ Solve the following differential equation $(D^2+4)y =$

$$\sin 2x \text{ given that when } x = 0 \text{ then } y = 0 \text{ and } \frac{dy}{dx} = 2.$$

- ✓ Solve $(D^3-1)y = xe^x + \cos^2 x$.

- ✓ Solve $(x^2 D^2 + xD - 4)y = x^2$.

1993

- ✓ Show that the system of confocal conics

$$\frac{x^2}{a^2+1} + \frac{y^2}{b^2+1} = 1 \text{ is self orthogonal.}$$

- ✓ Solve $\left\{ y \left(1 + \frac{1}{x} \right) + \cos y \right\} dx + \{ x + \log x - x \sin y \} dy = 0$.

- ✓ Solve $\frac{d^2 y}{dx^2} + w_0^2 y = a \cos wt$ and discuss the nature of solution as $w \rightarrow w_0$.

- ✓ Solve $(D^4 + D^2 + 1)y = e^{-\frac{x}{2}} \cos \left(\frac{\sqrt{3}x}{2} \right)$.

1994

- ✓ Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$.

- ✓ Show that if $\frac{1}{Q} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)$ is a function of x only

say, $f(x)$, then $F(x) = e^{\int f(x) dx}$ is an integrating factor of $Pdx + Qdy = 0$.

- ✓ Find the family of curves whose tangent form angle $\frac{\pi}{4}$ with the hyperbola $xy = c$.
- ✓ Transform the differential equation $\frac{d^2 y}{dx^2} \cos x + \frac{dy}{dx} \sin x - 2y \cos^3 x = 2 \cos^5 x$ into one having z an independent variable where $z = \sin x$ and solve it.
- ✓ If $\frac{d^2 x}{dt^2} + \frac{g}{b}(x-a) = 0$, (a , b and g being positive constants) and $x = a'$ and $\frac{dx}{dt} = 0$ when $t=0$, show that $x = a + (a' - a) \cos \sqrt{\frac{g}{b}} t$.
- ✓ Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$, where, $D = \frac{d}{dx}$.

1995

- ✓ Solve $(2x^2 + 3y^2 - 7)xdx - (3x^2 + 2y^2 - 8)y dy = 0$.
- ✓ Test whether the equation $(x+y)^2 dx - (y^2 - 2xy - x^2) dy = 0$ is exact and hence solve it.
- ✓ Solve $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$. (1998)
- ✓ Determine all real valued solutions of the equation $y''' - iy'' + y' - iy = 0$, $y' = \frac{dy}{dx}$.
- ✓ Find the solution of the equation $y'' + 4y = 8 \cos 2x$ given that $y = 0$ and $y' = 2$ when $x = 0$.

1996

- ✓ Solve $x^2(y - px) = yp^2$; $\left(p = \frac{dy}{dx} \right)$.
- ✓ Solve $y \sin 2x dx - (1 + y^2 + \cos^2 x) dy = 0$.
- ✓ Solve $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 10y + 37 \sin 3x = 0$. Find the value of y when $x = \frac{\pi}{2}$, if it is given that $y=3$ and $\frac{dy}{dx} = 0$ when $x=0$.
- ✓ Solve $\frac{d^4 y}{dx^4} + 2 \frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} = x^2 + 3e^{2x} + 4 \sin x$.
- ✓ Solve $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = x + \log x$.

1997

- ✓ Solve the initial value problem $\frac{dy}{dx} = \frac{x}{x^2 y + y^3}$, $y(0)=0$.
- ✓ Solve $(x^2 - y^2 + 3x - y) dx + (x^2 - y^2 + x - 3y) dy = 0$.
- ✓ Solve $\frac{d^4 y}{dx^4} + 6 \frac{d^3 y}{dx^3} + 11 \frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} = 20e^{-2x} \sin x$.
- ✓ Make use of the transformation $y(x) = u(x) \sec x$ to obtain the solution of $y'' - 2y' \tan x + 5y = 0$; $y(0)=0$; $y'(0) = \sqrt{6}$.
- ✓ Solve $(1+2x)^2 \frac{d^2 y}{dx^2} - 6(1+2x) \frac{dy}{dx} + 16y = 8(1+2x)^2$; $y(0) = 0$ and $y'(0) = 2$.

1998

- ✓ Solve the differential equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$.
- ✓ Show that the equation $(4x+3y+1) dx + (3x+2y+1) dy = 0$ represents a family of hyperbolas having as asymptotes the lines $x+y = 0$; $2x+y+1=0$. (1992)
- ✓ Solve the differential equation $y = 3px + 4p^2$.
- ✓ Solve $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}(x^2 + 9)$.
- ✓ Solve the differential equation $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = x \sin x$.

1999

- ✓ Solve the differential equation $\frac{xdx + ydy}{xdy - ydx} = \left(\frac{1 - x^2 - y^2}{x^2 + y^2} \right)^{\frac{1}{2}}$.
- ✓ Solve $\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 2y = e^x + \cos x$.
- ✓ By the method of variation of parameters solve the differential equation $\frac{d^2 y}{dx^2} + a^2 y = \sec(ax)$.

2000

- ✓ Show that $3 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} - 8y = 0$ has an integral

which is a polynomial in x . Deduce the general solution. 12

- ✓ Reduce $\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Qy = R$, where P, Q, R are functions of x , to the normal form.

Hence solve $\frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$. 15

- ✓ Solve the differential equation $y = x - 2a p + ap^2$. Find the singular solution and interpret it geometrically. 15

- ✓ Show that $(4x+3y+1)dx + (3x+2y+1)dy = 0$ represents a family of hyperbolas with a common axis and tangent at the vertex. 15

- ✓ Solve $x \frac{dy}{dx} - y = (x-1) \left(\frac{d^2 y}{dx^2} - x + 1 \right)$ by the method of variation of parameters. 15

2001

- ✓ A continuous function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = \begin{cases} 1 + e^{-t}, & 0 \leq t < 1 \\ 2 + 2t - 3t^2, & 1 \leq t \leq 5 \end{cases}$$

If $y(0) = -e$, find $y(2)$. 12

- ✓ Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log_e x$. 12

- ✓ Solve $\frac{dy}{dx} + \frac{y}{x} \log_e y = \frac{y(\log_e y)^2}{x^2}$. 15

- ✓ Find the general solution of $ayp^2 + (2x-b)p - y = 0$, $a > 0$. 15

- ✓ Solve $(D^2+1)^2 y = 24x \cos x$ given that $y=Dy=D^2y=0$ and $D^3y = 12$ when $x = 0$. 15

- ✓ Using the method of variation of parameters, solve

$$\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x. \quad 15$$

2002

- ✓ Solve $x \frac{dy}{dx} + 3y = x^3 y^2$. 12

- ✓ Find the values of λ for which all solutions of

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - \lambda y = 0 \text{ tend to zero as } x \rightarrow \infty. \quad 12$$

- ✓ Find the value of constant λ such that the following differential equation becomes exact.

$$(2xe^y + 3y^2) \frac{dy}{dx} + (3x^2 + I e^y) = 0$$

Further, for this value of λ , solve the equation. 15

- ✓ Solve $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$. 15

- ✓ Using the method of variation of parameters, find the

$$\text{solution of } \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \sin x \text{ with}$$

$$y(0) = 0 \text{ and } \left(\frac{dy}{dx} \right)_{x=0} = 0. \quad 15$$

- ✓ Solve $(D-1)(D^2-2D+2)y = e^x$ where $D = \frac{d}{dx}$. 15

2003

- ✓ Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal. 12

- ✓ Solve $x \frac{dy}{dx} + y \log y = x y e^x$. 12

- ✓ Solve $(D^5-D)y = 4(e^x + \cos x + x^3)$, where $D = \frac{d}{dx}$. 15

- ✓ Solve the differential equation $(px^2 + y^2)(px + y) = (p+1)^2$ where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions. 15

- ✓ Solve $(1+x^2)y'' + (1+x)y' + y = \sin 2[\log(1+x)]$. 15

- ✓ Solve the differential equation $x^2 y'' - 4xy' + 6y = x^4 \sec^2 x$ by variation of parameters. 15

2004

- ✓ Find the solution of the following differential equation

$$\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x. \quad 12$$

- ✓ Solve $y(xy+2x^2y^2)dx + x(xy-x^2y^2)dy = 0$. 12

- ✓ Solve $(D^4 - 4D^2 - 5)y = e^x(x + \cos x)$. 15

- ✓ Reduce the equation $(px-y)(py+x) = 2p$ where $p = \frac{dy}{dx}$ to Clairaut's equation and hence solve it. 15
- ✓ Solve the differential equation $x^2 \frac{d^3 y}{dx^3} + 2x \frac{d^2 y}{dx^2} + 2 \frac{y}{x} = 10 \left(1 + \frac{1}{x^2} \right)$. 15
- ✓ Solve $(x+2) \frac{d^2 y}{dx^2} - (2x+5) \frac{dy}{dx} + 2y = (x+1)e^x$. 15
- ✓ Solve the following differential equation $(1-x^2) \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} (1+x^2)y = x$. 15
- 2005**
- ✓ Find the orthogonal trajectory of a system of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter. 12
- ✓ Solve $xy \frac{dy}{dx} = \sqrt{x^2 - y^2 - x^2 y^2 - 1}$. 12
- ✓ Solve the differential equation $(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)y = \frac{1}{x+1}$. 15
- ✓ Solve the differential equation $(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$, where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. 15
- ✓ Solve the differential equation $(\sin x - x \cos x)y'' - x \sin xy' + y \sin x = 0$ given that $y = \sin x$ is a solution of this equation. 15
- ✓ Solve the differential equation $x^2 y'' - 2xy' + 2y = x \log x, x > 0$ by variation of parameters. 15
- 2006**
- ✓ Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbolas $xy = c, c > 0$. 12
- ✓ Solve the differential equation $(xy^2 + e^{-1/x^3})dx - x^2 y dy = 0$. 12
- ✓ Solve $(1+y^2) + (x - e^{-\tan^{-1} y}) \frac{dy}{dx} = 0$. 15
- ✓ Solve the equation $x^2 p^2 + yp(2x+y) + y^2 = 0$ using the substitution $y = u$ and $xy = v$ and find its singular solution, where $p = \frac{dy}{dx}$. 15
- ✓ Solve the differential equation $(D^2 - 2D + 2)y = e^x \tan x$, where $D = \frac{d}{dx}$, by the method of variation of parameters. 15
- 2007**
- ✓ Solve the ordinary differential equation $\cos 3x \frac{dy}{dx} - 3y \sin 3x = \frac{1}{2} \sin 6x + \sin^2 3x, 0 < x < \frac{\pi}{2}$. 12
- ✓ Find the solution of the equation $\frac{dy}{y} + xy^2 dx = -4x dx$. 12
- ✓ Determine the general and singular solutions of the equation $y = x \frac{dy}{dx} + a \frac{dy}{dx} \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{-1/2}$ 'a' being a constant. 15
- ✓ Obtain the general solution of $[D^3 - 6D^2 + 12D - 8]y = 12 \left(e^{2x} + \frac{9}{4} e^{-x} \right)$, where $D = \frac{d}{dx}$. 15
- ✓ Solve the equation $2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^3$. 15
- ✓ Use the method of variation of parameters to find the general solution of the equation $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 2e^x$. 15
- 2008**
- ✓ Solve the differential equation $y dx + (x + x^3 y^2) dy = 0$. 12
- ✓ Use the method of variation of parameters to find the general solution of $x^2 y'' - 4xy' + 6y = -x^4 \sin x$. 12
- ✓ Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$. 15

- with $y(0) = 1$, $y'(0) = -1$. 15
- ✓ Solve the differential equation $x^3 y'' - 3x^2 y' + xy = \sin(\ln x) + 1$. 15
- ✓ Solve the equation $y - 2xp + yp^2 = 0$ where $p = \frac{dy}{dx}$. 15
- 2009**
- ✓ Find the Wronskian of the set of functions $\{3x^3, |3x^3|\}$ on the interval $[-1, 1]$ and determine whether the set is linearly dependent on $[-1, 1]$. 12
- ✓ Find the differential equation of the family of circles in the xy -plane passing through $(-1, 1)$ and $(1, 1)$. 20
- ✓ Find the inverse Laplace transform of $F(s) = \ln\left(\frac{s+1}{s+5}\right)$. 20
- ✓ Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}$, $y(0) = 1$. 20
- 2010**
- ✓ Consider the differential equation $y' = \alpha x$, $x > 0$ where α is a constant. Show that—
(i) if $\phi(x)$ is any solution and $\Psi(x) = \phi(x) e^{-\alpha x}$, then $\Psi(x)$ is a constant;
(ii) if $\alpha < 0$, then every solution tends to zero as $x \rightarrow \infty$. 12
- ✓ Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3x)y' = 0$ admits an integrating factor which is a function of $(x+y^2)$. Hence solve the equation. 12
- ✓ Verify that $\frac{1}{2}(Mx + Ny)d(\log_e(xy)) + \frac{1}{2}(Mx - Ny)d(\log_e(\frac{x}{y})) = M dx + N dy$
Hence show that—
(i) if the differential equation $M dx + N dy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;
- (ii) if the differential equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)y dx + f_2(xy)x dy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx - Ny \equiv 0$. 20
- ✓ Show that the set of solutions of the homogeneous linear differential equation $y' + p(x)y = 0$ on an interval $I = [a, b]$ forms a vector subspace W of the real vector space of continuous functions on I . what is the dimension of W ? 20
- ✓ Use the method of undetermined coefficients to find the particular solution of $y'' + y = \sin x + (1+x^2)e^x$ and hence find its general solution. 20
- 2011**
- ✓ Obtain the solution of the ordinary differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$. 10
- ✓ Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1 - \cos \theta)$, (r, θ) being the plane polar coordinates of any point. 10
- ✓ Obtain Clairaut's form of the differential equation $\left(x \frac{dy}{dx} - y\right) \left(y \frac{dy}{dx} + y\right) = a^2 \frac{dy}{dx}$. Also find its general solution. 15
- ✓ Obtain the general solution of the second order ordinary differential equation $y'' - 2y' + 2y = x + e^x \cos x$, where dashes denote derivatives w.r. to x . 15
- ✓ Using the method of variation of parameters, solve the second order differendifferential equation $\frac{d^2 y}{dx^2} + 4y = \tan 2x$. 15
- ✓ Use Laplace transform method to solve the following initial value problem: $\frac{d^2 x}{dt^2} - 2 \frac{dx}{dt} + x = e^t$, $x(0) = 2$ and $\left. \frac{dx}{dt} \right|_{t=0} = -1$ 15