

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

TEST SERIES (MAIN)-2014

Test Code: FULL TEST P-II(M) IAS / T-08

MATHEMATICS
by K. VENKANNA
The person with 14 years of Teaching Experience

FULL TEST P-II

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

- This question paper-cum-answer booklet has 52 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question, shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- Symbols/notations carry their usual meanings, unless otherwise indicated.
- All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name: Gowtham Patru

Roll No.: 005663

Test Centre: ORN

Medium: English

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them
Gowthu
Signature of the Candidate

I have verified the information filled by the candidate above
Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

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THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			08
	(b)			08
	(c)			—
	(d)			08
	(e)			—
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			07
	(b)			06
	(c)			06
	(d)			18
4	(a)			11
	(b)			11
	(c)			08
	(d)			11
5	(a)			08
	(b)			08
	(c)			06
	(d)			05
	(e)			06
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			22
	(b)			08
	(c)			10
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				<u>177/250</u>

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P.T.O.

SECTION-A

1. (a) Suppose that a and b are group elements. If $|b| = 2$ and $bab = a^4$, determine the possibilities for $|a|$. (10)

$$\text{Given } |b| = 2$$

$$\Rightarrow b^2 = e$$

$$\text{also } bab = a^4$$

$$\Rightarrow a^4 \cdot a^4 = (bab)(bab)$$

$$\Rightarrow a^8 = b a^2 b$$

$$\Rightarrow a^8 = b a^2 b$$

$$\Rightarrow a^8 \cdot a^8 = (b a^2 b)(b a^2 b)$$

$$\Rightarrow a^{16} = b a^4 b$$

$$\Rightarrow a^{16} = b(b a^5) b$$

$$\Rightarrow a^{16} = b^2 a^5 b$$

$$\Rightarrow a^{16} = a$$

$$\Rightarrow a^{15} = e$$

$$o(a) \mid 15 \Rightarrow o(a) = 1 \text{ or } 3 \text{ or } 5 \text{ or } 15$$

~~108~~ \Rightarrow possibilities of

$$1 \text{ or } 3 \text{ or } 5 \text{ or } 15$$

1. (b) Let R be a ring and let $M_2(R)$ be the ring of 2×2 matrices with entries from R . Explain why these two rings have the same characteristic. (10)

Let n be the characteristic of ring R .

$$\Rightarrow na = 0$$

If $A \in M_2(R) \Rightarrow A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d \in R$

$$\Rightarrow nA = n \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix}$$

$$\therefore nA = n \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix}$$

- 08 -

\therefore characteristic of $M_2(R)$ is defined by

$$nA = 0 \Rightarrow \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow na = 0 \text{ & } nb = 0 \text{ & } nc = 0, nd = 0$$

\therefore characteristic of R & $M_2(R)$ are same

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1. (c) Examine the convergence of $\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x}$. (10)

$$\int_0^{\infty} f(n) dx = \int_0^1 f(n) dx + \int_1^{\infty} f(n) dx$$

consider $\int_0^1 \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x}$

Let $f(x) = \frac{e^{-x}}{x}$

~~$g(x) = \frac{1}{x}$~~

~~$\lim_{n \rightarrow 0} \frac{f(n)}{g(n)} = \lim_{n \rightarrow 0} e^{-n} = e^{\infty}$~~

A finite value $\Rightarrow f(n) \not\sim g(n)$ behave similarly at $x=0$

But $\int_0^1 \frac{1}{n} dx$ is not convergent at $n=0$

$\Rightarrow \int_0^1 \frac{1}{n} dx$ is not convergent at $n=0$

$\therefore \int_0^1 f(n) dx$ is not convergent at $n=0$

~~CFT~~

~~Step 2~~ $\lim_{n \rightarrow \infty} h(n) = \lim_{n \rightarrow \infty} \frac{1 - (1+n)e^{-n}}{n(n+1)}$

~~$R(n) = \frac{1}{n}$~~

~~$\lim_{n \rightarrow \infty} R(n) = 0$~~

$\therefore \int_0^{\infty} f(n) dx$ is not convergent

1. (d) If $u+v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, and $f(z) = u+iv$ is an analytic function of $z = x+iy$, find $f(z)$ in terms of z . (10)

$$U = U+V = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x} ; \quad f(z) = u+iv \\ \Rightarrow i f(z) = iu - iv$$

$$\Rightarrow U_x = \frac{(e^{2y}+e^{-2y}-2\cos 2x)(4\cos 2x) - 8\sin^2 x}{(e^{2y}+e^{-2y}-2\cos 2x)^2} = (1+i) f(z) = (u-v) + i(u+v) \\ = U + iV$$

$$\Rightarrow U_x(z, 0) = \frac{8\cot 2z - 8}{(2-2\cot 2z)^2} = \frac{-2}{1-\cot 2z}$$

$$\Rightarrow U_y = -\frac{-2\sin 2x (2e^{2y}-2e^{-2y})}{(e^{2y}+e^{-2y}-2\cos 2x)^2}$$

$$\Rightarrow U_y(z, 0) = 0$$

Milne-Thompson's Method

$$\therefore (1+i) f(z) = \int U_x(z, 0) dz - i \int U_y(z, 0) dz + c'$$

$$= \int \frac{2}{2-\cot 2z} dz + c'$$

$$= \int \frac{-2}{2\sin^2 z} dz + c'$$

$$= - \int \csc^2 z dz + c'$$

$$(1+i) f(z) = \frac{1}{2} \cot 2z + c'$$

$$\Rightarrow f(z) = \frac{(1-i)}{4} \cot 2z + c$$

1. (e) There are five pumps available for developing five wells. The efficiency of each pump in producing the maximum yield in each well is shown in the table below. In what way should the pumps be assigned so as to maximise the overall efficiency?

		Efficiency Well				
		W ₁	W ₂	W ₃	W ₄	W ₅
Pump	P ₁	45	40	65	30	55
	P ₂	50	30	25	60	30
	P ₃	25	20	15	20	40
	P ₄	35	25	30	25	20
	P ₅	80	60	60	70	50

The given problem
is maximization
problem
we need to convert
it into minimization
problem
(10)

Assignment Method

Step 1 : subtracting smallest element of every row (column)
from all other elements

$$\begin{array}{ccccc}
 15 & 10 & 25 & 0 & 25 \\
 25 & 5 & 0 & 35 & 5 \\
 \Rightarrow & 10 & 5 & 0 & 5 & 25 \\
 15 & 10 & 10 & 5 & 0 \\
 30 & 10 & 10 & 20 & 0
 \end{array}$$

$$\begin{array}{ccccc}
 5 & 5 & 25 & 0 & 25 \\
 15 & 0 & 0 & 35 & 5 \\
 \Rightarrow & 10 & 0 & 0 & 5 & 25 \\
 5 & 0 & 10 & 5 & 0 \\
 20 & 5 & 10 & 20 & 0
 \end{array}$$

Assigning zeroes

By assigning zeroes in a way that only one assignment per row or column

In the above table, 5 assignments are done \Rightarrow optimality is reached.

Optimal assignment

$$P_1 \rightarrow W_4 \quad t = 30$$

$$P_2 \rightarrow W_3 \quad = 25$$

$$P_3 \rightarrow W_1 \quad = 25$$

$$P_4 \rightarrow W_2 \quad = 25$$

$$P_5 \rightarrow W_5 \quad = 50$$

$$\therefore \text{maximum efficiency} = \frac{30 + 25 + 25 + 25 + 50}{5} \\ = \underline{\underline{155}}$$

2. (a) Let \mathbf{R}^* be the group of nonzero real numbers under multiplication and let $H = \{x \in \mathbf{R}^* \mid x^2 \text{ is rational}\}$. Prove that H is a subgroup of \mathbf{R}^* . Can the exponent 2 be replaced by any positive integer and still have H be a subgroup?

(10)

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2. (b) Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β . What are the possibilities for β if $\beta \in S_9$ (08)

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2. (c) (i) Is every finite set open? Justify your answer by giving an example.
(ii) Is the union of an arbitrary collection of closed sets closed? Justify your answer by an example.

(iii) Give an example of a family $\{I_n : n \in \mathbb{N}\}$ of non-empty closed intervals such that

$$I_1 \supset I_2 \supset I_3 \supset \dots \text{ and } \bigcap_{n=1}^{\infty} I_n = \emptyset.$$

Set

(iv) Let $S = \left\{ \frac{1}{n} / n \in \mathbb{N} \right\} \cup \left\{ 1 + \frac{3}{2n} / n \in \mathbb{N} \right\} \cup \left\{ 6 - \frac{1}{3n} / n \in \mathbb{N} \right\}$. Find derived set S' of S . Also find supremum of S and greatest number of S . (3+3+3+5=14)

Ex:-

$$\left\{ x \in \mathbb{R} / x \geq n \right\}.$$

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2. (d) Evaluate $\int_0^\pi \frac{a d\phi}{a^2 + \sin^2 \phi}$.

(18)

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3. (a) Let $R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Z}_7 \right\}$ with the usual matrix addition and multiplication and mod

7 addition and multiplication of the entries. Prove that R is commutative ring. How many elements are in R ? Is R a field? What happens when \mathbb{Z}_7 is replaced by \mathbb{Z}_5 ?

(15)

$$R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Z}_7 \right\}$$

$$\text{Let } A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}; B = \begin{bmatrix} c & -d \\ d & c \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} c & -d \\ d & c \end{bmatrix}$$

$$= \begin{bmatrix} ac - bd & -ad - bc \\ ad + bc & ac - bd \end{bmatrix}$$

~~Q~~ Also $BA = \begin{bmatrix} c & -d \\ d & c \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

~~$= \begin{bmatrix} ac - bd & -ad - bc \\ ad + bc & ac - bd \end{bmatrix}$~~

~~please
try to consider
try~~
 $\therefore AB = BA$

~~$\therefore R$ is a commutative Ring.~~

$$\text{as } a, b \in \{0, 1, 2, 3, 4, 5, 6\}$$

$\therefore a, b$ both can be chosen in 7 ways.

\therefore There are 49 elements in R

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in R$
and is multiplicative identity of $R[\mathbb{Z}_7]$

~~R is also~~ \mathbb{Z}_7 is a field as all non zero
elements have inverses

We can also find inverse for all non
zero elements of $R[\mathbb{Z}_7]$

$\therefore R[\mathbb{Z}_7]$ is a field.

$R[\mathbb{Z}_5]$ is also a field with
25 elements.

3. (b) What derangement of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ will reduce its sum to $\frac{1}{2} \log 2$?

7)

The series can be arranged as

$$S_n = 1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \dots$$

$$= (1 - \frac{1}{2} + \frac{1}{4}) + (\frac{1}{3} - \frac{1}{6} - \frac{1}{8}) + \dots$$

$$= (1 + \frac{1}{3} + \frac{1}{5} + \dots - \frac{1}{2n-1}) - (\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2n})$$

$$= (1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{2n}) - (\frac{1}{2} + \frac{1}{4} + \dots - \frac{1}{2n})$$

$$= (1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{2n}) - \frac{1}{2} (1 + \frac{1}{2} + \dots - \frac{1}{n})$$

$$= \frac{1}{2} (1 + \frac{1}{2} + \dots - \frac{1}{2n}) - \frac{1}{2} (1 + \frac{1}{2} + \dots - \frac{1}{n})$$

$$S_n = \frac{1}{2} (1 + \frac{1}{2} + \dots - \frac{1}{2n}) - \frac{1}{2} (1 + \frac{1}{2} + \dots - \frac{1}{n})$$

we know that $1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{n} = \gamma_n + \log n$

$$\therefore S_n = \frac{1}{2} (\gamma_{2n} + \log 2n) - \frac{1}{2} (\gamma_n + \log n)$$

$$= \frac{1}{2} (\gamma_{2n} - \gamma_n) + \frac{1}{2} \log 2$$

^{as $n \rightarrow \infty$} $S = \frac{1}{2} (\gamma - \gamma) + \frac{1}{2} \log 2$

^{206 =>} $S = \frac{1}{2} \log 2$

3. (c) Show that the series for which $S_n(x) = \frac{nx}{1+n^2x^2}$, $0 \leq x \leq 1$ cannot be differentiated term by term at $x=0$. (08)

$$S_n(x) = \frac{nx}{1+n^2x^2}$$

$$\lim_{n \rightarrow \infty} S_n(x) = \lim_{n \rightarrow \infty} \frac{nx}{1+n^2x^2} = \lim_{n \rightarrow \infty} \frac{x(1/n)}{\frac{1}{n^2} + x^2} = 0$$

$$\therefore \frac{d}{dx} \left[\lim_{n \rightarrow \infty} S_n(x) \right] = 0$$

$$\text{Now, } S_n'(x) = \frac{n}{1+n^2x^2} - \frac{nx(2n^2x)}{(1+n^2x^2)^2}$$

$$= \frac{1-n^3x^2}{(1+n^2x^2)^2}$$

$$\Rightarrow S_n'(0) = 1$$

$$\therefore \lim_{n \rightarrow \infty} S_n'(0) = 1$$

$$\therefore [S(0)] \neq \lim_{n \rightarrow \infty} S_n'(0)$$

$\therefore S_n(x)$ cannot be differentiate
term by term at $n=0$

3. (d) Use Simplex method to solve the following LPP.

$$\text{Maximise } Z = 6x_1 + 4x_2$$

$$\text{subject to } 2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0.$$

Obtain an alternative optimum BFS if it exists.

(20)

Standard form

$$\text{max } Z = 6x_1 + 4x_2 + 0\cdot b_1 + 0\cdot b_2 + 0\cdot b_3$$

$$2x_1 + 3x_2 + b_1 = 30$$

$$3x_1 + 2x_2 + b_2 = 24$$

$$x_1 + x_2 - b_3 = 3$$

Adding an artificial variable y_3 to the third constraint

Big-M method

C_B	y_3	x_B	y_1	y_2	y_3	y_4	y_5	y_6	
0	y_3	30	2	3	1	0	0	0	
0	y_4	24	3	2	0	1	0	0	
-M	y_6	3	(1)	1	0	0	-1	1	→
	(Z_j)		-M	M	0	0	M	-M	
	$(Z_j - C_i)$		-M-6	-M-4	0	0	M	0	

C_B	y_3	x_B	y_1	y_2	y_3	y_4	y_5	
0	y_3	24	0	1	1	0	2	
0	y_4	15	0	-1	0	1	(3)	→
6	y_1	3	1	1	0	0	-1	
	(Z_j)		6	6	0	0	-6	
	$(Z_j - C_i)$		0	2	0	0	6	

c_B	y_1	y_2	y_3	y_4	y_5	
0	14	0	$\frac{5}{3}$	1	$-\frac{2}{3}$	0
0	5	0	$-y_3$	0	y_3	1
6	8	1	$\frac{2}{3}$	0	y_3	0
(Z_j)		6	4	0	2	0
$(Z_j - L_j)$		0	0	0	2	0

optimality reached

$$\Rightarrow x_1 = 8 \quad \& \quad x_2 = 0$$

$$\underline{\max (z) = 48}$$

alternative optimal BFS can be obtained by removing y_3 from basic solution and adding y_2

c_B	y_1	y_2	y_3	y_4	y_5	
4	$\frac{4}{5}$	0	1	$\frac{3}{5}$	$-\frac{2}{5}$	0
0	$\frac{39}{5}$	0	0	$\frac{1}{5}$	$\frac{1}{5}$	1
6	$\frac{12}{5}$	1	0	$-\frac{2}{5}$	$\frac{3}{5}$	0
(Z_j)		48	6	4	0	2
$(Z_j - L_j)$		0	0	0	2	0

\therefore alternative solution $x_1 = \frac{12}{5}, x_2 = \frac{42}{5}$

4. (a) Find an integer n that shows that the rings \mathbb{Z}_n need not have the following properties that the ring of integers has.
- $a^2 = a$ implies $a = 0$ or $a = 1$.
 - $ab = 0$ implies $a = 0$ or $b = 0$.
 - $ab = ac$ and $a \neq 0$ imply $b = c$.
- Is the n you found prime? (12)

Let us consider \mathbb{Z}_{10} .

$$\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(i) $a^2 = a$; let $a = 5$

$$5^2 = 25 \pmod{10} = 5$$

$\therefore a^2 = a$; But ~~$a \neq 0 \neq 1$~~

(ii) $ab = 0$; let $a = 2, b = 5$

$$2 \cdot 5 = 10 \pmod{10} = 0$$

~~$= 0$~~

~~∴ $ab = 0$, But $a \neq 0 \neq b \neq 0$~~

(iii) $ab = ac$; let $a = 2, b = 3, c = 8$

$$2 \cdot 3 = 6$$

$$2 \cdot 8 = 16 \pmod{10} = 6$$

and $a \neq 0$

But $3 \neq 8$

$\therefore ab = ac \neq a \neq 0$, But ~~$b \neq c$~~

$\therefore z_n$ need not have above properties
that z holds.

The $n = 10$ we found is not prime
Because if n is a prime
 z_n holds the above properties.

4. (b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable on \mathbb{R} but f' is not continuous on \mathbb{R} . (13)

$$f(x) = x^2 \sin \frac{1}{x^2} ; f(0) = 0$$

continuity of $f(x)$ at $x=0$

$$\begin{aligned} \text{Let } f(x) &= \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x^2} \\ &= 0 \quad \left[\because \text{as } \sin \frac{1}{x^2} \rightarrow [-1, 1] \right. \\ &\quad \left. \text{as } x \rightarrow 0 \right] \end{aligned}$$

$\therefore f(n)$ is continuous at $n=0$

Differentiability at $n=0$

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin \frac{1}{h^2}}{h} \\ &= \lim_{h \rightarrow 0} h \sin \frac{1}{h^2} \\ &= 0 \quad \left[\because \sin \frac{1}{h^2} \text{ is finite as } h \rightarrow 0 \right] \end{aligned}$$

$\therefore f(n)$ is differentiable at $n=0$

Also $f(n)$ is definable at all $n \neq 0$ on R

$\therefore f(n)$ is differentiable on R

$$\text{Now, } f'(n) = \begin{cases} 2n \sin \frac{1}{n^2} - \frac{2}{n} \cos \frac{1}{n^2} & ; n \neq 0 \\ 0 & ; n=0 \end{cases}$$

Continuity of $f'(n)$ at $n=0$

$$\begin{aligned} \lim_{n \rightarrow 0} f'(n) &= \lim_{n \rightarrow 0} \left(2n \sin \frac{1}{n^2} - \frac{2}{n} \cos \frac{1}{n^2} \right) \\ &= -\infty \quad \left[\because \sin \frac{1}{n^2}, \cos \frac{1}{n^2} \text{ are finite as } n \rightarrow 0 \right] \end{aligned}$$

But $f'(0) = 0$

$\therefore f'(n)$ is not continuous at $n=0$

4. (c) Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ in powers of z ; where (i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$. (12)

$$f(z) = \frac{z+3}{z(z-1)(z+2)}$$

$$= -\frac{3}{2z} + \frac{4}{3(z+1)} + \frac{5}{6(z-2)}$$

$$(i) \quad |z| < 1 \Rightarrow \frac{|z|}{2} < 1$$

$$\therefore f(z) = -\frac{3}{2z} - \frac{4}{3}(1+z)^{-1} - \frac{5}{12}(1-\frac{z}{2})^{-1}$$

$$= -\frac{3}{2z} + \frac{4}{3}(1-z+z^2+\dots) - \frac{5}{12}(1+\frac{z}{2}+\frac{z^2}{4}+\dots)$$

$$(ii) \quad 1 < |z| < 2$$

$$\Rightarrow \frac{1}{|z|} < 1 \quad \text{and} \quad \frac{|z|}{2} < 1$$

~~$$f(z) = -\frac{3}{2z} - \frac{4}{3z}(1+\frac{1}{z})^{-1} - \frac{5}{12}(1-\frac{z}{2})^{-1}$$~~

~~$$= -\frac{3}{2z} - \frac{4}{3z}(1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\dots)$$~~

~~$$- \frac{5}{12}(1+\frac{z}{2}+\frac{z^2}{4}+\dots)$$~~

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$$(iii) |z| > 2$$

$$\Rightarrow \frac{1}{|z|} < 1 \text{ and } \frac{2}{|z|} < 1$$

$$\therefore f(z) = -\frac{3}{2z} - \cancel{\left(\frac{4}{3z} \left(1 + \frac{1}{z}\right)^{-1} \right)} + \frac{5}{6z} \left(1 - \frac{2}{z}\right)^{-1}$$

$$= -\frac{3}{2z} - \frac{4}{3z} \left(1 - \frac{1}{z} + \frac{1}{z^2} + \dots\right)$$

$$+ \frac{5}{6z} \left(1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \dots\right)$$

4. (d) Make a graphical representation of the set of constraints of the following LPP. Find the extreme points of the feasible region. Finally, solve the problem graphically.

$$\text{Maximise } Z = 2x_1 + x_2$$

$$\text{subject to } x_1 + x_2 \geq 5$$

$$2x_1 + 3x_2 \leq 20$$

$$4x_1 + 3x_2 \leq 25$$

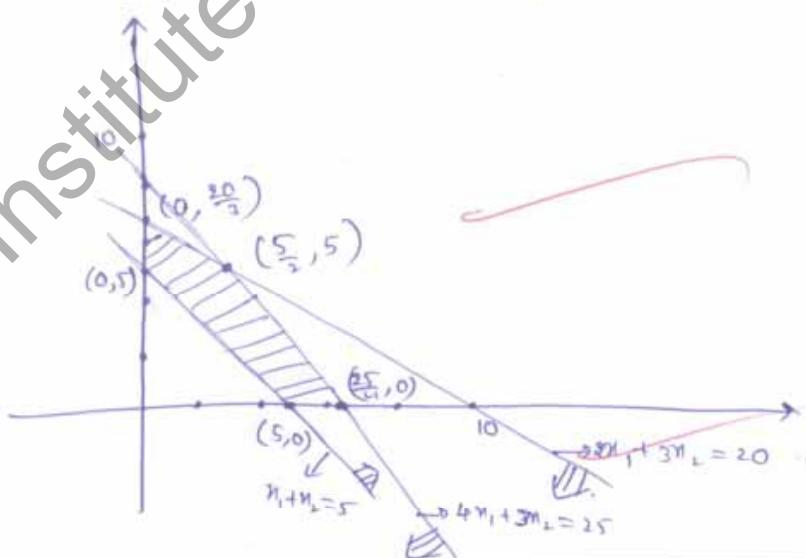
$$x_1, x_2 \geq 0$$

$$x_1 + x_2 \geq 5$$

$$\frac{x_1}{10} + \frac{x_2}{20/5} \leq 1$$

$$\frac{x_1}{25/4} + \frac{x_2}{25/3} \leq 1$$

(13)



From the above graph

The boundary points ~~are~~ of feasible region
 $(0, 5); (5, 0); \left(\frac{25}{4}, 0\right); \left(\frac{5}{2}, 5\right); \left(0, \frac{20}{3}\right)$

To maximise $Z = 2x_1 + x_2$

at

$(0, 5)$

value of Z

5

$(5, 0)$

10

$\left(\frac{25}{4}, 0\right)$

$\frac{25}{4}$

(maximum)

$\left(\frac{5}{2}, 5\right)$

10

$\left(0, \frac{20}{3}\right)$

$\frac{20}{3}$

The optimal solution is at

$$x_1 = \frac{25}{4}, x_2 = 0$$

optimal value $Z = \frac{25}{2}$

SECTION-B

5. (a) Solve $x^2 p^2 + y^2 q^2 = z^2$.

(10)

$$\text{Given } x^2 p^2 + y^2 q^2 = z^2$$

$$\Rightarrow \left(\frac{x}{z} \frac{\partial z}{\partial x}\right)^2 + \left(\frac{y}{z} \frac{\partial z}{\partial y}\right)^2 = 1 \rightarrow \textcircled{1}$$

$$\text{Let } \frac{1}{x} \partial x = \partial X ; \frac{1}{y} \partial y = \partial Y ; \frac{1}{z} \partial z = \partial Z$$

$$\Rightarrow X = \log x ; Y = \log y ; Z = \log z$$

$\textcircled{1}$ becomes

$$\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 = 1$$

$$\Rightarrow P^2 + Q^2 = 1 \quad \textcircled{2}$$

By charpit's method solution of $\textcircled{2}$ will be

$$P = a \quad \& \quad Q = b \Rightarrow Q = \sqrt{1-a^2}$$

$$\therefore dZ = P dx + Q dy$$

$$\Rightarrow dZ = adx + \sqrt{1-a^2} dy$$

$$\therefore Z = ax + \sqrt{1-a^2} y + C$$

$$\Rightarrow \log z = a \log x + \sqrt{1-a^2} \log y + \log C$$

$$\Rightarrow Z = C x^a \cdot y^{\sqrt{1-a^2}}$$

5. (b) Find a surface satisfying $r - 2s + t = 0$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$. (10)

Given Partial Differential Equation

$$(D^2 - 2DD' + D'^2) z = 6 \quad \rightarrow ①$$

C.F. $(D - D')^2 = 0$

P.T. $z_1 = \phi_1(y+x) + x\phi_2(y+x)$

$$\begin{aligned} \frac{1}{(D^2 - 2DD' + D'^2)} 6 &= \frac{1}{D^2 \left(1 - \frac{2DD' + D'^2}{D^2}\right)} 6 \\ &= \frac{1}{D^2} \left(1 - \frac{2DD' + D'^2}{D^2}\right)^{-1} 6 \\ &= \frac{1}{D^2} (1 + \dots) 6 \\ &= \frac{1}{D^2} 6 = \underline{3x^2} \end{aligned}$$

-08

\therefore solution of ④

$$z = \phi_1(y+x) + x\phi_2(y+x) + 3x^2 \rightarrow ②$$

Given ② touches $z = xy \rightarrow ③$
along $y=x$

$\therefore p$ & q are same for ② & ③ along $y=x$

$$p = \phi_1'(y+x) + \phi_2(y+x) + x\phi_2'(y+x) + 6x = y = x$$

$$\Rightarrow q = \phi_1'(y+x) + x\phi_2'(y+x) = x$$

$$\Rightarrow \phi_2'(y+x) + 6x = 0 \Rightarrow \phi_2'(y+x) = -6x$$

$$\Rightarrow \phi_2(x) = -6x$$

$$\Rightarrow \phi_2'(x) = -6$$

$$\Rightarrow \phi_2'(n) = -3$$

$$\Rightarrow \phi_2'(2n) = -3$$

$$\therefore \phi_1'(2n) = 6n^2 + 4n$$

$$\Rightarrow \phi_1'(x) = \frac{3}{2}x^2 + \frac{4}{2}$$

$$\Rightarrow \phi_1(x) = \frac{x^3}{3} + \frac{x^2}{4} + C$$

$$\Rightarrow \phi_1'(x) = 2x$$

$$\Rightarrow \phi_1(n) = n^2 + C$$

$$\therefore z = \frac{(x+y)^3}{3} + \frac{(x+y)^2}{4} + C = 3(x+y)x + 3y^2$$

$$\frac{(x+y)^3}{3} + \frac{(x+y)^2}{4} - 3xy + C$$

along $x=y$; above curve is ③ are same

$$z = (x+y)^2 - 3xy + C$$

$$\text{along } x=y; x^2 = n^2 + C \Rightarrow C=0$$

$$\therefore z = x^2 + y^2 - 3xy$$

5. (c) The current i in an electric circuit is given by

$i = 10e^{-t} \sin 2\pi t$ where t is in seconds. Using Newton's method, find the value of t correct to 3 decimal places for $i = 2$ amp. (10)

for $i = 2$ amp

$$10e^{-t} \sin 2\pi t = 2 \Rightarrow f(t) = 5e^{-t} \sin 2\pi t - 1 = 0$$

$$f(0) = -1 < 0$$

$$f(1) = 5e^{-1} \sin(\frac{\pi}{2}) - 1 = 5e^{-1} - 1 > 0$$

$\therefore f(t)$ has a solution in $(0, 1)$

$$\text{Let } t_0 = 0$$

$$f(t) = 5e^{-t} \sin 2\pi t - 1$$

$$\Rightarrow f'(t) = 10\pi e^{-t} \cos 2\pi t - 5e^{-t} \sin 2\pi t$$

Newton's method

$$t_{n+1} = t_n - \frac{f(t_n)}{f'(t_n)}$$

$$t_1 = t_0 - \frac{5 \sin 2\pi t_0 - e^{t_0}}{(10\pi \cos 2\pi t_0 - 5 \sin 2\pi t_0)}$$

$$t_0 = 0$$

$$\Rightarrow t_1 = 0.03183$$

$$t_2 = 0.06415$$

$$t = 0.03183$$

5. (d) (i) Realize the following expression by using NAND gates only.

$$g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e)f$$

where \bar{x} denotes the complement of x .

- (ii) Find the decimal equivalent of $(357.32)_8$

(10)

(i)

Given

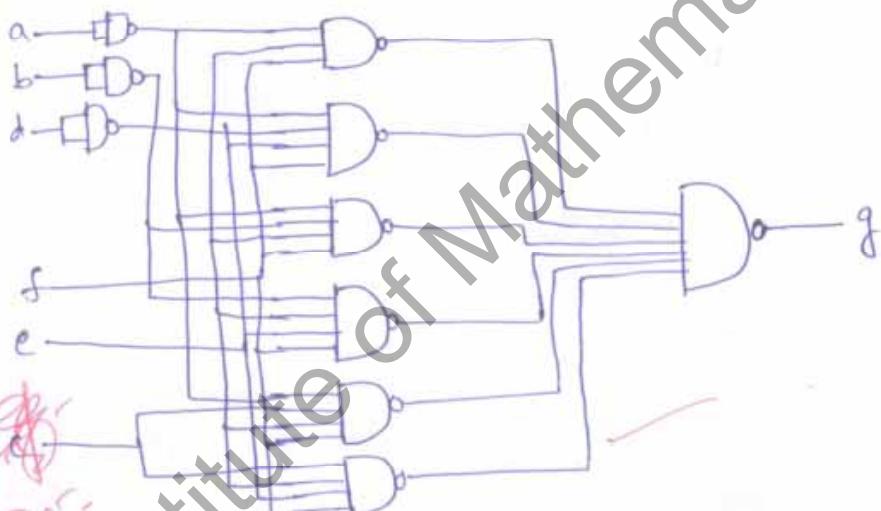
$$g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e)f$$

$$= (\overline{a \cdot b \cdot c}) \cdot (\bar{d}) \cdot (\overline{a \cdot e}) \cdot f$$

$$= (\bar{a}\bar{d}f + \bar{b}\bar{d}f + c\bar{d}f)(\bar{a}+e)$$

$$= \bar{a}\bar{d}f + \bar{a}\bar{d}ef + \bar{a}\bar{b}\bar{d}f + \bar{b}\bar{d}f + \\ \bar{a}c\bar{d}f + c\bar{d}ef$$

\therefore using NAND gates



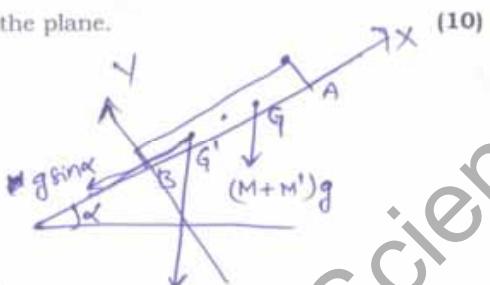
Q.S.
for better
solution
refer key.

$$\begin{aligned}
 (357.32)_8 &= 3(8^2) + 5(8^1) + 7(8^0) + 3(8^{-1}) + 2(8^{-2}) \\
 &= 3(64) + 5(8) + 7 + \frac{3}{8} + \frac{2}{64} \\
 &= 196.40625
 \end{aligned}$$

$\therefore \underline{\underline{192 + 40 + 7 + \frac{3}{8} + \frac{2}{64}}} = 239.40625$

5. (e) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M' , starting from the upper end, walks down the plank so that it does not move, show that he gets to the other end in time

$$\sqrt{\left\{ \frac{2M'a}{(M+M')g \sin \alpha} \right\}}, \text{ where } a \text{ is the length of the plane.}$$



initially

Man at A

\therefore Centre of gravity at G

$$\Rightarrow BG = \frac{M'a + M(\frac{a}{2})}{M+M'}$$

finally

Man at B'

\therefore centre of gravity

$$\Rightarrow BG' = \frac{M'a}{M+M'}$$

$$\therefore GG' = \frac{M'a}{M+M'}$$

considering the motion of centre of gravity

along the inclined plane

$$GG' = \frac{1}{2}(g \sin \alpha) t^2 \quad [\because \text{initial velocity is zero}]$$

$$06 \Rightarrow t^2 \left(\frac{g \sin \alpha}{2} \right) = \frac{M'a}{(M+M')}$$

$$\Rightarrow t = \sqrt{\frac{2M'a}{(M+M')g \sin \alpha}}$$

6. (a) Solve $(D + D' - 1)(D + D' - 3)(D + D')z = e^{x+y} \sin(2x + y)$

(12)

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6. (b) Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane

$$y = x.$$

(13)

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6. (c) The following table gives the velocity v of a particle at time t :

t (seconds) : 0 2 4 6 8 10 12

v (m/sec) : 4 6 16 34 60 94 136

Find the distance moved by the particle in 12 seconds and also the acceleration at $t=2$ sec.

(10)

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6. (d) Solve the equations $27x + 6y - z = 85$; $x + y + 54z = 110$; $6x + 15y + 2z = 72$ by Gauss-Seidal method. (15)

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7. (a) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = 10$ and $y = 10$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 10) = x(10 - x)$ while the other three faces are kept at 0°C . Find the steady state temperature in the plate.

(25)

~~At steady state~~

Lagrange's equation

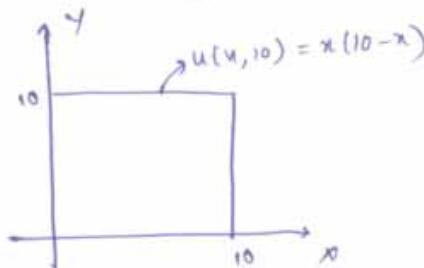
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\text{Let } u(x, y) = X(x)Y(y)$$

$$\Rightarrow \frac{X''}{X} + \frac{Y''}{Y} = 0 \quad \Rightarrow \frac{X''}{X} = -\frac{Y''}{Y} = \lambda \quad (\text{let})$$

Boundary conditions

$$X(0) = 0 ; X(10) = 0 ; Y(0) = 0$$



case (i)(let $\mu = 0$)

$$\Rightarrow x(n) = an + b \Rightarrow a = b = 0$$

$$\Rightarrow x(0) = 0$$

so, we reject $\mu = 0$ case (ii)(let $\mu = \lambda^2$)

$$\Rightarrow x(n) = ae^{\lambda n} + be^{-\lambda n} \Rightarrow a + b = 0$$

$$\cancel{ae^{\lambda n} + be^{-\lambda n} = 0}$$

$$\Rightarrow a = b = 0$$

$$\Rightarrow x(n) = 0$$

so, we reject $\mu = \lambda^2$ case (iii)(let $\mu = -\lambda^2$)

$$\Rightarrow x(n) = A \cos \lambda n + B \sin \lambda n$$

$$x(0) = 0 \Rightarrow A = 0$$

$$x(l) = 0 \Rightarrow \sin \lambda l = 0 \Rightarrow \lambda l = n\pi$$

$$\Rightarrow \lambda = \frac{n\pi}{l}$$

$$\Rightarrow y(y) = C e^{\lambda y} + D e^{-\lambda y}$$

$$\Rightarrow y_n(y) = C_n e^{\frac{n\pi y}{l}} + D_n e^{-\frac{n\pi y}{l}}$$

$$y_n(0) = 0 \Rightarrow C_n + D_n = 0 \Rightarrow D_n = -C_n$$

$$\therefore y_n(y) = C_n \left[e^{\frac{n\pi y}{l}} - e^{-\frac{n\pi y}{l}} \right]$$

$$= 2C_n \sinh \frac{n\pi y}{l}$$

$$\text{also } x_n(n) = B_n \sin \frac{n\pi n}{l}$$

$$\therefore u_n(n, y) = E_n \sin \frac{n\pi n}{l} \sinh \frac{n\pi y}{l} \quad \text{where } E_n = 2B_n C_n$$

$$\Rightarrow u(x, y) = \sum_{n=1}^{\infty} E_n \sin \frac{n\pi x}{10} \sinh \frac{n\pi y}{10}$$

We know $u(x, 10) = x(10-x)$

$$\Rightarrow x(10-x) = \sum_{n=1}^{\infty} E_n \sinh n\pi \sin \frac{n\pi x}{10}$$

Fourier transform

$$E_n \sinh n\pi = \frac{2}{10} \int_0^{10} (10x - x^2) \sin \frac{n\pi x}{10} dx$$

$$\Rightarrow E_n \sinh n\pi = \left[-\frac{(10x - x^2) \cos \frac{n\pi x}{10}}{n\pi/10} + \int (10 - 2x) \frac{\cos \frac{n\pi x}{10}}{(n\pi/10)^2} dx \right]$$

$$= \left[-\frac{(10x - x^2) \cos \frac{n\pi x}{10}}{n\pi/10} + \frac{(10 - 2x) \sin \frac{n\pi x}{10}}{(n\pi/10)^2} - \frac{2 \cos \frac{n\pi x}{10}}{(n\pi/10)^3} \right]_0^{10}$$

$$= \left[-\frac{-2 \cos n\pi}{(n\pi/10)^3} + \frac{2}{(n\pi/10)^3} \right]$$

$$= \frac{2 \cdot 10}{n^3 \pi^3} [1 - (-1)^n]$$

$$E_n = \frac{400}{n^3 \pi^3 \sinh n\pi} [1 - (-1)^n]$$

$$E_{2m-1} = \frac{800}{n^3 \pi^3 \sinh (2m-1)\pi}$$

$$u(x, y) = \sum_{m=1}^{\infty} \frac{800}{n^3 \pi^3 \sinh (2m-1)\pi} \sin \frac{(2m-1)\pi x}{10} \sinh \frac{(2m-1)\pi y}{10}$$

7. (b) Using modified Euler's method, find an approximate value of y when $x = 0.3$, given that $dy/dx = x+y$ and $y = 1$ when $x = 0$. (10)

$$\frac{dy}{dx} = x+y \Rightarrow f(x, y) = x+y$$

$$y(0) = 1 \quad ; \quad h = 0.15$$

modified Euler's method

$$y(0.15) = y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 1 + (0.15)(0+1) = 1.15$$

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$= 1 + \frac{0.15}{2} [0+1 + 0.15+1.15]$$

$$= 1.1725$$

$$y_1^{(2)} = 1 + \frac{0.15}{2} [0+1 + 0.15+1.1725]$$

$$= 1.174$$

$$y_1^{(3)} = 1 + \frac{0.15}{2} [0+1 + 0.15+1.174]$$

$$= 1.1743$$

$$y(0.15) = 1.1743$$

$$y_2^{(0)} = y_1 + h f(x_1, y_1)$$

$$= 1.1743 + (0.15)(0.15+1.1743)$$

$$= 1.3729$$

$$y_2^{(1)} = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_2, y_2^{(0)})]$$

$$= 1.3729 + \frac{0.15}{2} [0.15+1.1743+0.3+1.3729]$$

$$= 1.3991$$

$$y_2^{(2)} = 1.1743 + \frac{0.15}{2} [0.15 + 1.1743 + 0.3 + 1.3991]$$

$$= 1.4010$$

$$y_2^{(3)} = 1.1743 + \frac{0.15}{2} [0.15 + 1.1743 + 0.3 + 1.4010]$$

$$= 1.4012$$

~~∴ $y(0.3) = 1.4012$~~

7. (c) For the given set of data points

$(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_n, f(x_n))$

write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula.

Lagrange's interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} f(x_1) + \frac{(x-x_1)(x-x_3)\dots}{(x_2-x_1)(x_2-x_3)\dots} f(x_2) + \dots + \dots$$

$$= \sum_{m=1}^n f(x_m) \cdot \prod_{\substack{h=m \\ h \neq m}}^n \frac{(x-x_h)}{(x_m-x_h)}$$

Algorithm

Step1: Enter x_1, x_2, \dots, x_n

Step2: Enter $f(x_1), f(x_2), \dots, f(x_n)$

Step3: Enter $x ; m=1 ; A=0$

Step 4 : ~~If~~ $m \leq n$ (or) Go to Step 5
Go to Step 12

Step 5 : $A = 1$

Step 6 : ~~If~~ $n < m$ Go to Step 7 (or)
Step 7 : ~~If~~ $n = m$ Skip Step 8 Go to Step 11

Step 8 : $K = \frac{\pi}{2} \frac{(x - x_n)}{(x_m - x_n)}$

Step 9 : $A = A + K \cdot f(x_m)$

Step 10 : $n = n + 1$; ~~REPEAT~~

Step 11 : $m = m + 1$

Step 12 : Print A

8. (a) Determine the motion, of a spherical pendulum, by using Hamilton's equations.

(16)

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8. (b) A uniform lamina is bounded by a parabolic arc, of latus rectum $4a$, and a double ordinate at a distance b from the vertex. If $b = \frac{1}{3}a(7 + 4\sqrt{7})$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there. (18)

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8. (c) A source of fluid situated in space of two dimensions is of such strength that $2\pi\mu$ represents the mass of fluid of density ρ emitted per unit of time. Show that the force necessary to hold a circular disc at rest in the plane of source is $2\pi\rho\mu^2 a^2 / r(r^2 - a^2)$, where a is the radius of the disc and r the distance of the source from its centre. In what direction is the disc urged by the pressure? (16)

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$$\therefore \frac{1 - \left(\frac{1+\gamma}{2}\right)e^{-\gamma}}{\omega(1+\gamma)\omega}$$

①

$$\frac{1 - \left(\frac{1+\gamma}{2}\right)e^{-\gamma}}{\left(\frac{a}{r} + \frac{1+\gamma}{2}\right)}$$

$\frac{1 - e^{-\gamma}}{\frac{a}{r}}$

$\frac{1}{r}$

1 -

END OF THE EXAMINATION

ROUGH SPACE

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ROUGH SPACE

$$\begin{aligned}
 & \left(1 + \frac{1}{2} + \frac{1}{n}\right) + \left(\frac{1}{3} + \frac{1}{8} - \frac{1}{8}\right) \\
 & \quad \left(1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{2n-1}\right) \\
 & \quad - \left(\frac{1}{2} + \frac{1}{n} + \dots - \frac{1}{4n}\right) \\
 & \left(1 + \frac{1}{2} + \dots - \frac{1}{2n}\right) - \frac{1}{2} \left(1 + \frac{1}{2} + \dots - \frac{1}{n}\right) \\
 & \quad \frac{1}{2} \left(1 + \frac{1}{2} + \dots - \frac{1}{2n}\right) \\
 & \quad \frac{1}{2} \left[2_{2n} + \log 2n - \frac{1}{2n} + \log n\right] \\
 & \quad \frac{1}{2} \left[2_{2n} + \log 2n - \frac{1}{2n} + \log n\right] \\
 & \quad \left(1 + \frac{1}{2} + \dots - \frac{1}{2n}\right) \\
 & \quad \left(1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{4n-1}\right) \\
 & \quad \left(1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{2n-1}\right) \\
 & \quad - \frac{1}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots - \frac{1}{4n}\right) \\
 & \quad \left(1 + \frac{1}{2} + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \dots - \frac{1}{n}\right) \\
 & \quad 2_{2n} + \log 2n - 2_n - \log n + 2_{4n} + \log 4n - \frac{1}{2} (2_{2n} + \log 2n) \\
 & \quad \quad \quad - \frac{1}{2} (2_n + \log n) \\
 & \quad \quad \quad \log \frac{4n}{2n \cdot 2n} = \log \frac{(2^2)^{3/2}}{2} \\
 & \quad \quad \quad = \frac{3}{2} \log 2
 \end{aligned}$$

ROUGH SPACE

$$m = \frac{M \frac{\alpha}{2} + M' \alpha}{M + M'}$$

$$\delta = \frac{M' \alpha}{M + M'}$$

$$\delta = \frac{1}{2} g t^2$$

$$m' = \frac{M \frac{\alpha}{2}}{M + M'}$$

0.1
 0.1335
 0.1626
 0.2024
 0.2380
 0.2744
 0.0970
 0.1304
 1.645
 1992
 234
 271
 30.8
 34.6
 38.5
 42.6
 46.2

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix} = \begin{bmatrix} * & 3 \\ 4 & * \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} * & -b \\ a & a \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 1 & 0 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} 1 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & a \\ b & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \bar{d} + \begin{bmatrix} 0 & 1 \\ 6 & 0 \end{bmatrix} (\bar{a} + e) \quad \bar{a}\bar{d}f + \bar{a}\bar{b}\bar{d}f + \bar{a}e\bar{d}f$$

$$\begin{array}{l} a+b+c=1 \\ a+e=0 \\ \cancel{a+b+c=0} \\ \cancel{a+e=0} \end{array} \quad \begin{array}{l} \frac{b+d=1}{b+6d=0} \\ \cancel{\frac{b+6d=0}{n^2}} \cdot \frac{-2}{n^2} \end{array} \quad \begin{array}{l} + \bar{a}\bar{d}e \\ d = -\frac{6}{5} \\ = 4 \end{array}$$

$$\frac{b=3}{a=4} \quad -\frac{2}{n} \cancel{\frac{c}{n^2}} \cdot \frac{1}{n} = 5 \quad n_1 = 5/2$$

$$\begin{bmatrix} 1 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 3 & 4 \end{bmatrix} \quad 5 + 3n_2 = 20 \quad n_2 = 5$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \frac{1}{15} - \frac{2}{15}$$

$$\frac{5b=1}{b=1/5} \quad 8 - \frac{28}{5} \quad \frac{72 + 168}{5}$$

$$\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \quad \frac{12}{5} \quad \begin{bmatrix} a+4b \\ b-a \end{bmatrix} \quad \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \\ \left\{ \begin{array}{l} a+b=1 \\ a-b=0 \end{array} \right. \quad a=1 \quad = \quad \boxed{}$$

OUR ACHIEVEMENTS from 2008 to 2013

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Total Marks in Maths (Opt.)

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